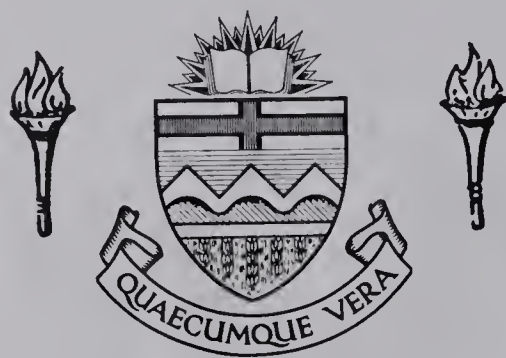


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FIRST GRADE MATHEMATICS ACHIEVEMENT AND
CONSERVATION

BY



ABRAM JOHN REIMER

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "First Grade Mathematics Achievement and Conservation," submitted by Abram John Reimer, in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of this study was to investigate the effect of certain factors upon achievement in first grade mathematics. Attention was focused particularly on the concept of conservation and its relationship to mathematics achievement.

A sample of eighty-one children was selected at random from eight grade one classrooms within the public school system of Edmonton, Alberta. The age, sex, father's occupation, kindergarten/playschool attendance, and Detroit Beginners First Grade Intelligence Test scores for each child were obtained from the school records.

Each subject was involved in two separate testing situations in the spring of 1968. The first was an individual conservation test consisting of five subtests designed specifically for this study. The mathematics achievement test, Seeing Through Arithmetic Test 1, was administered to each class by the classroom teacher.

The data thus collected were analyzed. First a correlational analysis was carried out to investigate the relationship of each of the variables to conservation and also to mathematics achievement. Secondly partial correlation coefficients were calculated and a suitable t test applied in order to assess the extent of correlation between two variables when the differential effects of a third variable

were removed. Finally Hotelling's t test statistic was used to determine whether differences in certain correlations were significant.

The major results of the study were:

(1) With first grade children, neither conservation or mathematics achievement is significantly related to the variables of age, sex, socio-economic status, or kindergarten/ playschool attendance.

(2) A significant relationship exists between intelligence and the ability to conserve.

(3) A significant relationship exists between mathematics achievement and intelligence both before and after the differential effects of conservation are removed.

(4) A significant relationship exists between mathematics achievement and conservation but knowledge of the ability to conserve does not contribute significantly more to prediction of mathematics scores if intelligence scores are known. However, the pupils who were conservers of number, quantity, and length were, with few exceptions, also high scorers on the mathematics achievement test.

(5) The correlations of different kinds of conservation, such as number, quantity, and length, with mathematics achievement are not significantly different.

The study concluded with several implications for education and some suggestions for further research.

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CHAPTER I

THE PROBLEM

I. INTRODUCTION

Educators are constantly searching for ways and means of improving the educational process so that more children will experience success in learning. This search has encompassed the areas of the scope and sequence of curriculum, as well as methodology. Fundamental research into the thought processes of children is of real significance to any educator concerned with improvement.

For some time it has been recognized that children even of the same age are different and vary in abilities. But trying to determine why or how they are different is not an easy task. In fact, it is not even clear what tasks the child can perform at various ages or at what stage of development it is most productive or beneficial to confront the individual with certain learning situations. The questions of what, when, and how are still largely unanswered in the educational curriculum field. Researchers, such as Jean Piaget, are making it possible for educators to supply some more answers for these important questions.

In the area of elementary school mathematics it is well known that within every group of children at a certain

age or grade level, some experience more difficulty and frustration than others. There are those who begin at the grade one level as "mathematical failures" and remain in that classification for life. Why is it that children of the same age and with the same educational opportunities vary so much in achievement? Saying that general intelligence and pre-school experiences make the difference may be an answer, but the answer is too general to be of much help.

Piaget's studies of child development reveal some factors that may be very significant to the learning of mathematics. In his view the concept of conservation or invariance is one of the fundamental requirements for mathematical learning (Piaget, 1952, pp. 3-4). That is to say, unless a child realizes that certain characteristics of an object or group of objects remain invariant under certain transformations it is impossible for him to understand related numerical concepts.

The purpose of this study was to investigate the effect of various factors upon mathematics achievement. Special attention was given to the concept of conservation and its relationship to achievement in first grade mathematics.

II. THE NULL HYPOTHESES

Achievement in first grade mathematics was studied in relation to the factors of chronological age, sex, socio-economic status, kindergarten/playschool attendance, intelligence, and conservation. Special attention was given to the concept of conservation. The following null hypotheses were tested:

(1) There is no significant relationship between conservation and the variables of chronological age, sex, socio-economic status, or kindergarten/playschool attendance.

(2) There is no significant relationship between conservation and intelligence.

(3) There is no significant relationship between mathematics achievement and the variables of chronological age, sex, socio-economic status, or kindergarten/playschool attendance.

(4) There is no significant relationship between mathematics achievement and intelligence.

(5) There is no significant relationship between mathematics achievement and intelligence when the differential effects of conservation are eliminated.

(6) There is no significant relationship between mathematics achievement and conservation.

(7) There is no significant relationship between

mathematics achievement and conservation when the differential effects of intelligence are eliminated.

(8) The correlations of different kinds of conservation with mathematics achievement are not significantly different.

III. SIGNIFICANCE OF THE STUDY

The mathematics curriculum within our schools is constantly under scrutiny by teachers, supervisors, curriculum committees, and the public. At the same time, publishing companies are producing printed and manipulative materials which they hope will be used in the classrooms. Anyone involved with the mathematics program must consider the abilities and development of the child as well as the scope and sequence of the concepts to be presented. The teacher must consider the development and needs of each individual as he attempts to operate within the framework of the mathematics program.

It is hoped that this study will throw some additional light on the question of readiness by indicating whether the concept of conservation is related to success in the mathematics program. If such a relationship exists, the study should also indicate which areas of conservation are most crucial to the program. Such information could influence the readiness testing program of the school, the age for beginning formal instruction of various topics of

a mathematical nature, and the content of the mathematics program.

Some benefit should also be derived by the classroom teacher for understanding and helping the child who is finding it difficult to cope with the mathematics materials. In addition, the results may indicate whether further attempts should be made to accelerate the development of children, specifically in the area of conservation.

Thus, it is the hope of the investigator that this study will prove valuable in providing educators with data which will aid in designing courses and materials which will more adequately meet the needs of children at various stages of development.

IV. LIMITATIONS OF THE STUDY

In interpreting the data of this study, the following limitations should be kept in mind.

(1) No effort was made to obtain a measure of validity for the conservation test, although an estimate of reliability was calculated. It is assumed that the test measures those concepts that it was intended to measure.

(2) The mathematical achievement test (STAT 1) may not differentiate between rote learning and meaningful learning. Piaget indicates that conservation is a prerequisite for mathematical understanding.

(3) No exhaustive attempt was made to study or control all the factors that affect conservation and/or achievement. This study will be limited to some of the aspects that are considered fundamental to meaningful learning of grade one mathematics.

(4) The sample was restricted to urban children using the grade one Seeing Through Arithmetic program.

(5) No attempt was made to control variables such as teacher effectiveness and availability of instructional materials.

(6) No effort was made to determine the length of time that a child had possessed a given kind of conservation. Further, it is not known how long a child needs to possess conservation before it will affect achievement.

V. DEFINITIONS OF SPECIAL TERMS

Conservation is defined as the logical understanding that invariance of quantity is maintained despite deformations or spatial displacements of objects, sets or subsets of objects, or any other matter.

Conservation of number refers to the conservation of a set of discrete objects such as blocks.

In this study, conservation of quantity refers to the conservation of continuous matter such as water or semi-continuous matter such as puffed wheat. Such

quantities are usually measured in terms of certain containers.

Conservation of length refers to the conservation of the linear size of objects like sticks or the paths along which we walk.

A state is a particular condition or configuration of an object, set of objects, or other matter exhibiting a particular magnitude or amount.

A transformation is the process of changing one state into another state.

One-to-one correspondence is the process of matching one element of a set with an element of another set such that a one-to-one relationship exists between the two elements.

Readiness is defined as the state of being prepared to gain meaning from a particular kind of instruction.

Mathematics achievement is a measure of mathematical knowledge and understanding acquired to a large degree from an instructional program. In this study this measure will be expressed in terms of the STAT 1 scores.

STA refers to the Seeing Through Arithmetic series published by Gage and Company.

STAT 1 refers to the Seeing Through Arithmetic Test published by Scott Foresman and Company for use at the end of the STA 1 course.

VI. THE EXPERIMENTAL SETTING

The experimental design is reported in detail in Chapter III. A brief overview is given here for the purpose of orientation.

A sample of eighty-one grade one children was selected at random from three schools in the Edmonton Public School System. A conservation test made up of a battery of five subtests, each containing two items, was designed by the investigator and administered to each subject during the month of May, 1968. Most of the test items were patterned after those used by Piaget in his investigations of conservation. The relationships of chronological age, sex, socio-economic status, kindergarten/playschool attendance, and intelligence to conservation ability and to mathematics achievement were analyzed. The main analysis involved a close examination of the relationship of conservation to mathematics achievement.

VII. OUTLINE OF THE REPORT

Having introduced the problem and the study in this chapter, a review of the literature related to the problem will be presented in Chapter II. Chapter III will include a detailed description of the experimental design and a discussion of the statistical procedures used in analyzing the data. The results of the data analysis will be

reported in Chapter IV. Finally, a summary of the study including the conclusions, implications, and suggestions for further research will be presented in Chapter V.

CHAPTER II

A REVIEW OF THE RELATED LITERATURE

The purpose of this chapter is to look generally at Piaget's developmental theory and specifically at the concept of conservation. Some of the studies and replications that have dealt with conservation are summarized and special mention is made of research that has attempted to discover the relationship between the ability to conserve and mathematics achievement.

I. PIAGET'S DEVELOPMENTAL THEORY

Piaget's developmental theory is the result of many years of clinical work with children. Experiments in every major aspect of intellectual development have led to mutually supporting results. The starting-point of the child's intellectual growth is his own action.

It is action which becomes progressively internalized through the child's acquisition of language and his growing use of symbols, through imagination and representation; and which then goes on expanding under the guidance partly of social life, partly of the physical world, till it culminates in a great organized scheme of mental operations (Isaacs, 1960, p. 6).

Piaget has been interested in mental growth in general and particularly in the way in which children develop concepts of space, time, causality, and number. His method of study was to interview children individually, giving

them simple but highly ingenious tests to complete, and questioning them afterwards on the reasons for their responses. On the basis of many years of such work he distinguishes several stages or levels of development paralleled in most of the variables he investigated. The four major stages of cognition are the sensorimotor, the preoperational, the concrete operational, and the formal operational stages. It is not until the concrete operational stage, ages seven to eleven years, that the child is capable of conceptual thought and can integrate two temporally separate experiences into a single judgment.

Concept of Conservation

The child's ability to comprehend the principle of conservation as it applies to matter and quantity is, for Piaget, a landmark in the development of logical thinking. Prior to this achievement, the child's thought tends to be dominated by his perceptions. He reasons transductively from one particular instance to another particular instance and is often unaware when his conclusions contradict one another. Once conservation has been attained, thought becomes more conceptual. The child is less likely to be taken in by the appearance of things. He can deal with more complex relationships, not only taking into account the immediate situation but mentally making comparisons and exploring the similarities

and the differences in previous experiences.

All of these differences can be summed up in the statement that the older child has at his command "a coherent and integrated cognitive system with which he organizes and manipulates the world around him" (Flavell, 1963, p. 165). The information from his experiences is mentally registered in such a fashion that he can readily think his way through them, moving forward or backward at will. He has no difficulty cancelling out the effects of change in order to focus on the elements of an experience that have remained unchanged.

The younger child, who has not yet firmly established such a system and cannot so readily reverse the direction of his thought is more prone to errors in judgment. "Specifically, there is as yet no conservation which is the psychological criterion of the presence of reversible operations" (Piaget in Ripple and Rockcastle, 1964, p. 9). An example of this may be seen when a child is confronted with two identical glass containers with equal amounts of water. The child agrees that the two quantities are equal. When the water of one container is poured into a third glass container with a greater diameter, the non-conserver may reply that the amount is less now because the water level is lower. He is unable to reverse the action in his mind and thus recapture the initial equivalence of the two amounts of water.

Piaget contends that conservation is a necessary condition for all rational activity including mathematics.

This being so, arithmetical thought is no exception to the rule. A set or collection is only conceivable if it remains unchanged irrespective of the changes occurring in the relationships between the elements.... A number is only intelligible if it remains identical with itself, whatever the distribution of the units of which it is composed. A continuous quantity such as length or volume can only be used in reasoning if it is a permanent whole, irrespective of the possible arrangements of its parts ... in each and every case the conservation of something is postulated as a necessary condition for any mathematical understanding (Piaget, 1952, pp. 3-4).

In the field of mathematics, some of the necessary conditions would include the conservation of quantity, number, length, area, weight, and volume. Although variations exist and it is unwise to attach ages to these concepts, on the average quantity, number, and length are conserved by children of ages six to eight, while area, weight, and volume are conserved between the ages of nine to twelve. Studies that disagree significantly with these ages usually involve children with unusual backgrounds or they involve a different method of testing. Research reported by Olson (1967) and Piaget (Ripple and Rockcastle, 1964, pp. 31-32) indicates that it is possible that some children attain the concept of conservation several years later than the majority. Sawada (1966), on the other hand, lowered the attain-

ment age for conservation of length by using a non-verbal testing technique.

Too often in the past, the teaching of mathematics in the primary grades has been based on the premature assumption of number understanding. Piaget's work indicates that mathematics teaching should take the following order, and not the reverse, as has so often been the case:

- (1) Some things are stable and invariable.
- (2) These things can be grouped, and groups have the qualities of "one-ness", "two-ness", etc.
- (3) Number properties have an order of size.
- (4) These quantities are represented by certain symbols in our culture.

Conservation, then, is not the only prerequisite for meaningful number development although it is considered a basic and necessary one.

Lovell and Ogilvie, in their study of this concept, conclude:

The concept of conservation or invariance of substance is an important one. A quantity such as a lump of plasticine, a collection of beads, a length or a volume can be used by the mind only if it remains permanent in amount and independent of the rearrangement of its individual parts. This notion of invariance is indeed essential to any kind of measurement in the physical world (Lovell and Ogilvie, 1960, p. 109).

If conservation is deemed to be so necessary for the development of mathematical concepts, what is it? Ripple states:

Apparently to achieve success in a conservation task children need an internalized symbol system to insulate their thought processes from the misleading perception of the visual display (Ripple and Rockcastle, 1964, p. 58).

Piaget explains that:

The reasoning that leads to the affirmation of conservation essentially consists in coordination of relations, with its two-fold aspect of logical multiplication of relations and mathematical composition of parts and proportions (Piaget, 1952, p. 18).

Conservation may be defined as the logical understanding that invariance is maintained despite deformations or spatial displacements of objects, sets or subsets of objects, or continuous quantities.

The essential mental capabilities for conservation are reversibility, combination of compensated relations, and identity operations (Piaget, 1960(b), pp. 140-141). A conservation problem may be solved by using one or all of these processes. Referring again to the previously cited example of the liquid being poured from one of two identical containers to a container with a larger base, a conserver may reason in one of three ways. If he says that the amount of liquid in the larger container is still the same because nothing has been added or subtracted,

he is using the notion of identity. If he says it is still the same because the decrease in height compensates for the increase in width, he is using the combination of compensated relations. If, on the other hand, the conserver insists that the amount is still the same because the liquid could be poured back into the smaller container and would then reveal equality, he is using reversibility to solve the problem. The subject's rationale usually indicates which mental reasoning he utilized in a particular problem.

The development of the concept of conservation is generally divided into three stages. A child at the first stage, absence of conservation, fails to understand that invariance is maintained in spite of transformations or rearrangements of parts. During the intermediary stage, the child vacillates between conservation and non-conservation. He senses that contradictions exist but often his perceptions overrule his logical reasoning. Once the child has reached the final stage of conservation he is certain that features such as quantity remain invariant regardless of the perceptual changes that occur.

At the Conference on Cognitive Studies and Curriculum Development, Piaget described four factors which contribute to cognitive development (Ripple and Rockcastle, 1964, pp. 10-14). The first is maturational, the increasing differentiation of the nervous system due

to natural growth. The second is experience with the physical and logical-mathematical world. The third, social transmission, influences development as the child assimilates and accommodates information incidentally from his contacts with other human beings and systematically from various educational agencies. For Piaget, the fourth factor, equilibration, is fundamental and consists of active compensation or self-regulation leading to reversibility. Faced with an external disturbance, the child will react in order to compensate and consequently he will tend towards equilibrium. It is a process of self-regulation in which the individual is the active coordinator of his own development.

Although Piaget's main concern does not lie in the field of education, many applications to education are suggested by his system. The necessity for and method of diagnostic assessment of the child's ability or readiness is implied. Planning curricula with respect to content and grade placement in the light of Piaget's findings as well as adopting teaching methods in harmony with the knowledge gained from Piaget on how children internalize ideas are further applications that can be made. Piaget himself makes some pointed comments with respect to cognitive development and mathematics education. He states:

It is a mistake to suppose that a child acquires the notion of number and other mathematical concepts just from teaching.

On the contrary, to a remarkable degree he develops them himself, independently and spontaneously. When adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth.... In short, children must grasp the principle of conservation of quantity before they can develop the concept of number. Now conservation of quantity of course is not in itself a numerical notion; rather it is a logical concept. Thus these experiments in child psychology throw some light on the epistemology of the number concept (Piaget, 1953).

Having surveyed Piaget's system and especially the concept of conservation it is appropriate to summarize next some of the investigations and experiments that have dealt with conservation.

II. THE RELATION OF CONSERVATION TO AGE, EXPERIENCE, AND INTELLIGENCE

Many of Piaget's experiments have been replicated for the purpose of establishing the age at which children begin to conserve. Generally these agree with previous findings. Lunzer (1960, pp. 31-32) has suggested that simple conservation, classification, and seriation so often appear round about the same age, seven or eight, because the same psychological mechanisms are involved in all these concepts namely, the ability to compare two judgments simultaneously. When the child can hold two judgments in thought, simultaneously, he is liberated

from the here and now -- that is, from his perceptions.

Piaget agrees that the age at which children begin to conserve a particular quantity may vary by several years. Studies, such as Fornwald's (1964) and Pelletier's (1966), show that some children in grade one do not possess conservation of number or length. If the concept of invariance is as basic as Piaget suggests, then it is conceivable that some grade one students experience difficulties because they are expected to manipulate mathematical ideas before one of the necessary prerequisites, conservation, is present.

Goodnow and Bethon (1966) suggest that in the normal course of events, children acquire the skills they need for conservation tasks without benefit of schooling. They used unschooled Chinese, Europeans, and Americans and tested them on conservation of weight, volume, and area. Mermelstein and Shulman (1967) drew the same conclusion from their research using some Negro children in Virginia who did not have formal schooling. Generally, though, it is felt that the experiences a child has influence logical development.

That the ability to conserve is positively correlated with intelligence is accepted by many. Elkind (1961), in his study of quantitative thinking in which he replicated Piaget's experiments with sticks, liquids, and beads, found significant correlations with some of the WISC

subtests (Information - .47, Arithmetic - .35, Picture Arrangements - .55, Object Assembly - .38, Coding - .42). The correlation with the full scale I.Q. was .43 and with verbal performance .47.

Goodnow and Bethon (1966) contend that among school children all tasks show a close relation to mental age. Using the Stanford Binet test, Feigenbaum (1963) concludes that I.Q. makes a significant difference to conservation. Almy (1966, p. 47) goes one step further in emphasizing that the ability to conserve represents one dimension of the child's developing intellectual power.

Piaget clarifies his position on standardized intelligence tests when he states:

It is indisputable that these tests of mental age have on the whole lived up to what was expected of them; a rapid and convenient estimation of an individual's general level. But it is no less obvious that they simply measure a "yield" without reaching constructive operations themselves. As Pieron quite rightly pointed out, intelligence conceived in these terms is essentially a value-judgment applied to complex behaviour (Piaget, 1960(b), p. 153).

It may be noted then that regardless of whether conservation is a subset of intelligence or whether conservation is affected generally by intelligence, conservation tests should give a more precise and specific assessment of a child's psychological development than an intelligence test.

III. ACCELERATING CONSERVATION

Duckworth (in Ripple and Rockcastle, 1964, pp. 1-5) summarizes some of Piaget's ideas on the effect of schooling on cognitive development. Children go through consecutive stages but good pedagogy can have an effect on this development. Specific training on a conservation task makes little sense because it is not likely to have a general effect on understanding but, a broader approach can modify a child's effective set of mental operations. A delay in attainment of conservation or some other Piagetian concept is possibly due to a lack of questioning and struggling for solutions and little call for physical or intellectual activity.

Numerous attempts have been made to teach or accelerate conservation. These attempts may be classified as reinforce-practice methods, reversibility methods, and cognitive conflict methods. Wallach and Sprott (1964), Brison (1966), Wohlwill and Lowe (1962), and Beilen and Franklin (1962) were able to produce some improvement in conservation abilities using the first two methods in an experimental setting. On the whole the results were not very significant and researchers concluded that Piagetian concepts were inordinately difficult to stamp in.

However, Smedslund (1961) found that it was possible to induce conservation of substance in children by means

of a training procedure designed to induce cognitive conflict, which is, according to Smedslund, an essential condition for the development of conservation in the child. Cognitive conflict supposedly induces a reorganization of the child's intellectual actions which proceeds along the lines postulated by Piaget's equilibration theory. The reorganization then leads to the conservation strategy. Gruen (1965) was also able to produce some improvement in number conservation using the conflict approach. Towler (1967) reviews the attempts to develop conservation experimentally and concludes that the various methods have had limited success with no one method being completely successful for the purpose for which it was intended. In his own attempts he is quite successful in teaching conservation of continuous quantities to partial and non-conservers. Thus, especially for children who are near conservation or who have been retarded because of environmental factors, stimulating experiences are beneficial in developing basic concepts such as conservation.

IV. MATHEMATICAL ACHIEVEMENT

As was mentioned above, Piaget's view is that certain logical concepts are necessary before a grasp of numbers is possible. A child must be able to conserve number, produce a one-to-one correspondence, classify, and seriate in order to proceed successfully with a program involving

number operations.

Skemp (in Land, 1963, pp. 40-48) develops a three-part theory for learning mathematics.

- (1) Mathematics depends on reflective intelligence not just general intelligence.
- (2) Mathematical concepts are formed in a field of experiences but only when the necessary background is established beforehand.
- (3) Concepts or ideas need to be developed in some logical order.

Generally it is agreed that intelligence is correlated with mathematics achievement. Steffe (1966) noted that mean scores on a test of addition facts for middle and upper I.Q. groups were significantly higher than the mean score for the lower I.Q. group. Overholt (1964, p. 19) reports a study by Erickson which established a correlation of .73 between mathematics achievement and intelligence. Barakat (1951) states that general or innate intelligence is the most important factor underlying the mathematical abilities of grammar school pupils.

V. MATHEMATICAL ACHIEVEMENT AND CONSERVATION

From a review of related literature one can conclude that the child's level of development places a limit on what may be acquired by virtue of experience or training at a particular time. Certain logical concepts such as

conservation are necessary before a grasp of numbers is possible. Piaget contends that if the necessary pre-concepts are lacking, any instruction in mathematics will simply result in rote memorization of facts. Conservation, then, is necessary for meaningful learning or understanding of mathematical ideas.

Two experimental studies which indicate a need for further investigation have been carried out by Dodwell (1961) and Hood (1962). Dodwell constructed a test of number concepts -- conservation, correspondence, seriation, cardination, and ordination -- along Piagetian lines, but with greater objectivity than is possible using Piaget's "clinical" methods. Short-term stability (test-retest) correlation with an interval of one week was reasonably high at $r = .87$. When the scores of thirty-four kindergarten children on this test were compared with their subsequent performance in a simple teacher-made test of arithmetical achievement at grade one, six months later, the correlation was $.59$. Dodwell concludes that such concept tests would be useful arithmetic readiness tests for grade one entrants.

Hood (1962) devised tests of concept development involving conservation of number and quantity, correspondence, seriation, and additive composition of classes and numbers, again similar to some of Piaget's experiments. He compared the total scores of 126 children aged four to six years

with teachers' ratings of the arithmetical skills of their pupils and found a substantial degree of correspondence. Most of those who were successful in the Piagetian tests were found to be making reasonable progress in number work, while there were "no instances of success in arithmetic and failure in the Piaget tests being found in the same child" (Hood, 1962, p. 284).

The results of both of these investigations can be regarded only as starting points since Dodwell's group of subjects was rather small and Hood relied on teachers' ratings of arithmetical achievement rather than on an objective assessment of performance.

A group test was used by Overholt (1964) to determine the ability of fourth grade children to conserve quantity. He used two separate tests, the first of which involved three tasks with plasticine and the second involved four tasks with cotton rope. No significant differences were found between conservers and non-conservers in total arithmetic achievement means as measured by the Iowa Tests of Basic Skills when adjustments were made for differences in intelligence levels between the two groups. It should be noted, however, that this investigation assumed that conservation of quantity such as plasticine and cotton would be significantly related to the variety of mathematical concepts or abilities that the Iowa Test measures. One might conclude that the differences were

not significant because the type of conservation considered was not related to much of the material in the achievement test.

A somewhat extensive study by Almy (1966) into young children's thinking included a look at the relationship of conservation and achievement. The conservation test provided each child with seven opportunities to indicate whether or not he was conserving. The opportunities involved conservation of number using blocks and conservation of an amount of liquid. A cross-sectional study including forty-eight second-grade children from a lower class school did not indicate a significant correlation between the ability to conserve and premeasurement and numerical concepts as assessed by the New York Inventory of Mathematical Concepts. Although the results are not clearly reported, Almy concludes that generally "children who are able to conserve at an early age do better in other tests related to mental ability and to beginning reading and arithmetic" (Almy, 1966, p. 84).

In the longitudinal part of Almy's study, the sum of standard scores representing each child's performance in conservation at five successive interviews from kindergarten to grade two were correlated with the mathematics scores obtained in grade two. Although there were only forty-one children from the middle class school and twenty-four from the lower class school who remained in the study

for the five successive interviews, the correlations ranging from .26 to .53 indicate that some positive relationship exists between the ability to conserve and the ability to perform on a mathematical achievement test.

In Steffe's study (1966) a conservation test of numerosness was used to group grade one children into four levels. Children in the lowest level of conservation received significantly lower scores on a test of addition problems and addition facts than the children in the other three levels.

Steffe's conservation test consisted of twelve tasks in which the subject was asked to compare two sets of objects arranged so as to appear equal or unequal in number. Unlike Piaget's tasks, no one-to-one correspondence was used to establish equality and no transformations were performed with the objects. The children could make their decision on the basis of counting, on the basis of making a mental one-to-one correspondence between the two sets, or on the basis of perceptual clues. The effects of intelligence were controlled by placing all the subjects into three categories of intelligence. The mathematical performance of the subjects in the four conservation levels within each intelligence category was compared.

VI. SUMMARY

A survey of Piaget's theory reveals the necessity of possessing certain cognitive abilities before meaningful learning can take place in various areas. Ripple states:

Knowledge of the relationship, if one can be found, between the learner and what is to be learned -- between the cognitive abilities of children at different levels of development and the structure of the subject matter they are to learn -- should permit us to make enormous strides in increasing the effectiveness and efficiency of learning as it goes on in school situations (Ripple and Rockcastle, 1964, p. 50).

One of these cognitive abilities is the ability to conserve or to understand that invariance is maintained despite deformations or spatial displacements of objects, groups of objects, or continuous quantities. Different kinds of conservation such as number, length, and weight have been agreed upon but even within any one of these areas it is possible that smaller sub-areas exist. That is, a child may conserve small numbers but he may be led awry by his perceptions when the numbers become greater than ten.

Experiments that have dealt with ages of conservation show that many children do not yet possess this concept of invariance in areas that may be strategically related to the mathematics concepts presented in school.

Although the relationship between the ability to conserve and the ability to do mathematics has been studied, the results are inconclusive. Some investigators have limited themselves to one aspect of conservation and thus their findings do not indicate how some of the other possible aspects are related to achievement. Other studies are restrictive in that the sample is small or the achievement measure is unreliable or limited to one area.

The literature reviewed gives some idea of the need to undertake further studies involving conservation. There is a need to establish what the role of conservation is as a prerequisite to mathematical learning; and what kinds of conservation are most essential for the various topics included in the present mathematics program.

CHAPTER III

THE EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

This chapter contains an explanation of the selection of the sample, a description of the tests used including the procedure of administration, and a discussion of the statistical procedures used to analyze the results of the study.

I. THE SAMPLE

The sample was selected from the grade one classrooms of three elementary schools in the Edmonton Public School System. After eliminating one class which was on an experimental program, eight classes totalling 183 students remained for consideration. Two of these students were excluded because they were repeating the mathematics course. This left 181 students from which to select a sample for the study.

The sample of 83 subjects was chosen by assigning a number to each of the 181 students. A table of random numbers was consulted and eighty-three numbers were chosen (Kenney and Keeping, 1954). Since one student said she was moving before the study would be completed and another was absent during the administration of the conservation test, a total of eighty-one grade one students participated

in the study. The sample consisted of thirty-eight girls and forty-three boys. Of these, fifty-eight subjects or 71.6 per cent had attended playschool or kindergarten. Table I provides a further breakdown of the sample with respect to age, intelligence, and socio-economic status. There is no reason to suspect that this sample differs significantly from the total grade one population of a typical urban society in Alberta.

TABLE I

RANGE, MEAN, AND STANDARD DEVIATION OF AGE,
INTELLIGENCE, AND SOCIO-ECONOMIC STATUS

	Age in Months	I.Q.*	S.E.S.*
Range	77 - 94	63 - 143	38.8 - 78.8
Mean	83.4	102.4	50.7
S.D.	3.7	16.0	8.9

* See next section for source of I.Q. and S.E.S. scores.

II. INSTRUMENTATION

Data such as age, father's occupation, and I.Q. were obtained from the school records. For the purpose of analysis each subject's age was recorded in months as of June 1, 1968. The Blishen Occupational Class Scale (Blishen, 1961) was consulted and the appropriate numerical value representing the father's socio-economic status was assigned

to each subject. The Canadian mean in 1951 was fifty while the standard deviation was ten.

Each of the subjects had been given the Detroit Beginners First-Grade Intelligence Test during the month of September. The University of Alberta has established Edmonton norms for this test, based on the raw scores obtained in September, 1963. The revised scores used by the schools and quoted in this study are the result of applying the Edmonton revision scale.

Each subject was involved in two separate testing situations in the spring of 1968. The first session was an individual conservation test consisting of five subtests. The latter session was a group test of mathematical achievement administered by the classroom teacher.

The Conservation Test

Since no suitable instrument was available, a set of items largely patterned after tests used by Piaget in his investigations was developed. The items were grouped into five subtests identified as:

- (1) Conservation of Number Less Than Ten,
- (2) Conservation of Number Greater Than Ten,
- (3) Conservation of Number in an Additive
Rearrangement,
- (4) Conservation of Quantity,
- (5) Conservation of Length.

Although the STAT 1 includes only two questions directly related to quantity and length, these two concepts are emphasized and discussed in greater detail in the STA 1 text than the test indicates. It may also be argued that the concept of conservation is general; thus being able to conserve quantity or length indicates the presence of a basic cognitive ability which will benefit the subject in solving problems in many facets of a mathematical or scientific nature. For these reasons subtests four and five were included in the conservation test.

In order to check the suitability of the questions and to refine the instructions and procedures, the test was administered to a pilot group of eight grade one students who did not form part of the sample. On the basis of this pilot study several items were revised and the instructions were refined.

The revised form of the test (see Appendix A) consisted of five subtests with two items in each. A description of each subtest follows.

Subtest 1. Conservation of number less than ten.

This subtest was designed to measure the subject's ability to realize that a small group of objects retains the same numerical characteristic despite transformation of the objects. It was theorized that a child who was still

operating at the perceptual level and who lacked the concept of invariance would think that a group becomes greater when the members of the group are arranged so as to appear more than before.

The necessary apparatus consisted of seven blue blocks and seven red blocks, each one being a three-quarter-inch cube. After S* had placed the same number of red blocks in a row parallel to the seven blue blocks and had agreed that both rows had the same number of blocks, E* spread the red blocks into a longer row. The lower left arrangement in Figure 1 illustrates the pattern that confronted the subject when he was asked to give his response. (The questions for each item were similar and required S to compare two sets of objects, two quantities, or two lengths. See Appendix A.)

For the second item of this subtest the red blocks were moved into a close bunch as illustrated by the lower center arrangement in Figure 1. Again the subject was required to state whether there were more red blocks, more blue blocks, or the same number of blocks in each set.

* S - subject

* E - examiner

MATERIALS USED IN CONSERVATION TEST FOR ITEMS
1a (LOWER LEFT), 1b (LOWER CENTER),
2a (LOWER RIGHT), 2b (UPPER LEFT),
AND 3a (UPPER RIGHT)

This subtest was designed to measure the subject's ability to realize that a larger group of objects retains the same numerical characteristic despite transformations of the objects. Since it has been suggested that a conserver of small numbers may revert to non-conservation when working with larger amounts (Flavell, 1963, p. 359), the two items of this subtest involving groups of fifteen and twelve objects respectively were included.

For the first item, thirty wooden beads, two

identical small jars, and one larger jar were required. After the subject had dropped fifteen beads simultaneously into each of the two smaller jars, the beads from one of these jars were poured into the larger jar. The lower right arrangement of Figure 1 shows the beads in the two different-sized jars as they appeared to S for comparison purposes.

The apparatus for the second item of this subtest consisted of a twelve-inch square sheet of gray paper on which twelve yellow one-inch square pieces of paper and twelve blue one-inch square pieces of paper were arranged in two concentric circles having diameters of four inches and seven and one-half inches respectively. (See upper left portion of Figure 1.) This is the only item of the test in which the subject was not asked to agree on equivalence before the perceptual conflict was presented for evaluation. It was felt that the subject could establish this equivalence himself by noting that each yellow piece was in one-to-one correspondence with a blue piece. Situations like this which are classifiable as "unprovoked correspondence" were used exclusively by Steffe (1966) in his research on conservation.

Subtest 3. Conservation of number in an additive rearrangement. This subtest was designed to measure the subject's ability to recognize that the total number of a

given set of objects remains invariant when the elements of the set are rearranged into different-sized subsets. In order to answer the first item correctly S had to realize that the total in two subsets was equal to the total in two different sized subsets. In the second item it was necessary to realize that the total is equal to the sum of its parts.

The material for the first item consisted of two sheets of blue paper having dimensions of nine inches by twelve inches and sixteen Ritz crackers. While introducing the problem with a short story about between-meal snacks, E placed the eight crackers on each sheet in a four and four arrangement. After agreeing that there were the same number of crackers on each page, E rearranged the crackers on the second page into a one and seven arrangement. The upper right illustration in Figure 1 shows the visual problem that was presented to the subject when he was asked to compare the number of crackers on each page.

For the second item of this subtest S was asked to place the same number of red blocks in a row parallel to nine blue blocks which E had placed on the table. After agreeing on equivalence, the red blocks were rearranged into three subsets of four, two, and three blocks respectively and S was asked to compare the number of blocks of each color. In the following diagram the top row represents the blue blocks while the lower groups

represent the red blocks.

Blue -- 

Red -- 

Subtest 4. Conservation of quantity. This subtest was designed to measure the subject's ability to realize that a continuous quantity such as water and a semi-continuous quantity such as puffed wheat remains invariant when poured into different sized or shaped containers.

In the first item, two identical jars containing colored water were used. After the contents had been equalized to S's satisfaction, the water from one was poured into two smaller jars. The upper left illustration in Figure 2 shows the problem that faced S when he was asked to compare the amount of water in the large jar with the total amount in the two smaller jars.

For the second item of this subtest, puffed wheat was equalized in two larger jars and then the contents of one of these jars was poured into a smaller jar. As can be observed in Figure 2 (upper right), the larger jar was only about half full while the smaller jar was completely full when S was asked to compare the quantity of puffed wheat in the two different containers.

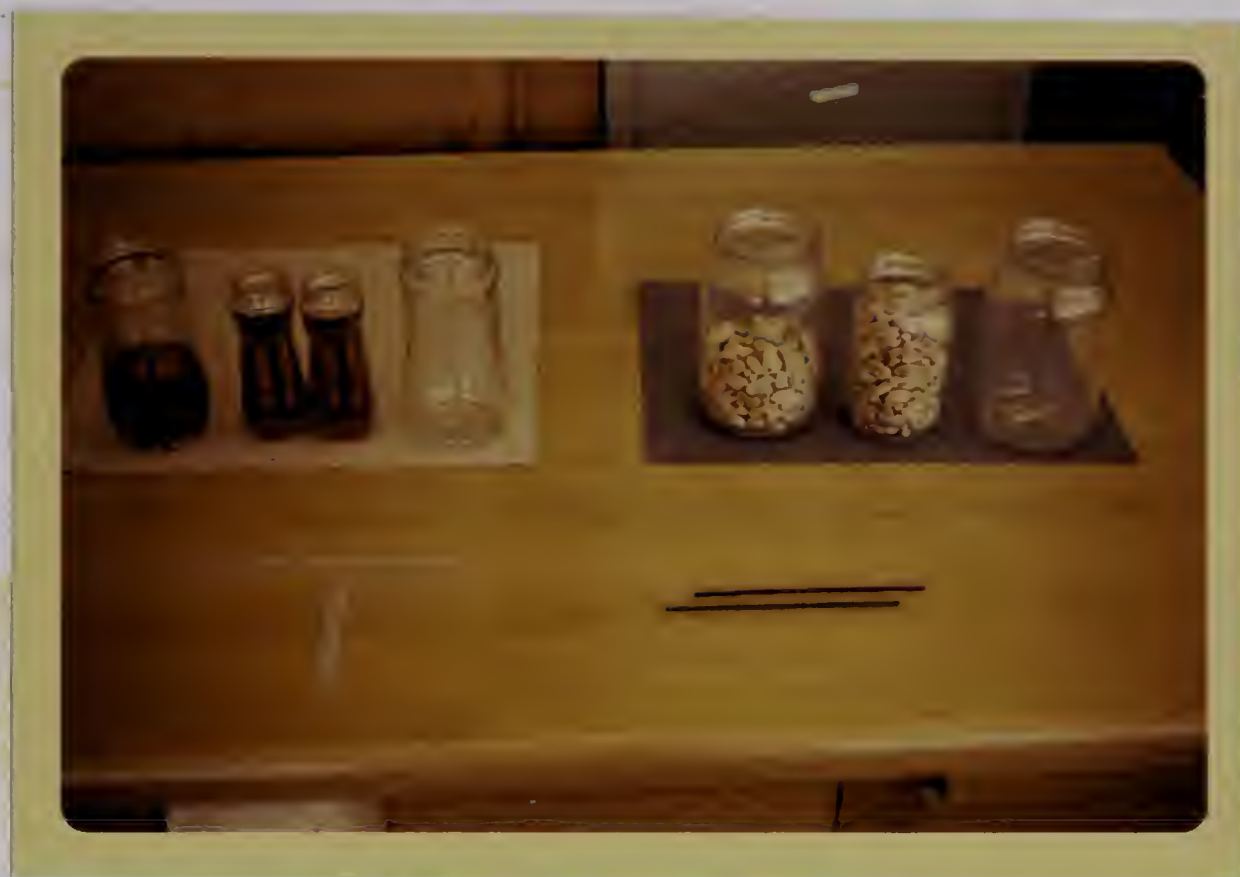


FIGURE 2

MATERIALS USED IN CONSERVATION TEST FOR ITEMS
4a (UPPER LEFT), 4b (UPPER RIGHT),
5a (LOWER LEFT), AND 5b (LOWER RIGHT)

Subtest 5. Conservation of length. This subtest was designed to test the ability to realize that the length of an object remains invariant despite subdivision and transformations.

In the first item, two identical strips of gray construction paper were placed before S with ends coterminous. The strip nearest to S had previously been cut near the middle at a forty-five degree angle. After S agreed that both "sidewalks" were the same length, the one nearest to the subject was "rebuilt" into a "sidewalk that goes around

a corner". Figure 2 (lower left) shows the two "side-walks" whose lengths S was asked to compare.

Two identical sticks, six inches in length, were used for the second item of this subtest. Once S had agreed that they were of equal length, the stick nearest S was moved about three-quarters of an inch to the left. Figure 2 (lower right) shows the position of the two sticks when S was asked whether one was longer than the other or whether they were both the same length.

Procedure. The conservation test was given individually by the investigator during the period of May 10th to 22nd, 1968. On the basis of random selection the members of the sample from each room were known. Before calling each subject to the testing room, the investigator was introduced to the children and spent a few minutes chatting with the class as a whole. The participants were assured that the time spent with the investigator would be quite enjoyable.

The apparatus was set up on a large table in a small "testing room". E was seated at the table while S stood beside him. The necessary materials were moved to a convenient position in front of S as they were required for each item of the test. (See Figure 3.)



FIGURE 3

A SUBJECT STANDING IN FRONT OF THE TESTING TABLE
(Posed after he had been tested.)

At the beginning of each session E verified some of the personal data that had been collected for each S. This was followed by an introductory item which was designed to introduce S to the general format of the test items. S was asked to place the same number of red blocks in a row parallel to the seven blue blocks which E had placed on the table. As E pointed to the appropriate rows he asked, "Are there more blue blocks, more red blocks, or the same number of blocks in each row?" After S had agreed that there were the same number in each row, E removed two red blocks and asked, "What about now?".

The two red blocks were then replaced and again E asked, "What about now?" E then proceeded with the test proper.

The entire session lasted about fifteen minutes.

Scoring. As the test proceeded, E circled S's response on an individual score sheet. (See Appendix B.) In 1a, for example, L was circled if S said the longer (red) row had more blocks, Sh if S said the shorter (blue) row had more, and S if S said they were both the same. Unusual reactions and interesting comments were noted on the score sheet as well. A correct answer was scored as one while an incorrect answer was scored zero. It was assumed that "the present state of knowledge about conservation does not permit a much finer scaling of responses than the categorical procedure of assigning ones and zeros" (Sawada, 1966, p. 61).

For those parts of the study in which the results of a single subtest are used, a one or a zero was given to the entire subtest. If S did both items of a subtest correctly he scored a one. If one or both of the answers were incorrect a score of zero was given. It was assumed that this method of scoring individual subtests would ensure that only those subjects who were definitely conservers would be labelled as such.

Controls built into the test. In order to ensure that S had understood the problem and also to discourage

guessing, S was asked to give a reason for each of his answers. If E had any reason to doubt the reliability of S's response, he would repeat or rephrase the question.

In a multiple choice type of question there is also the possibility that S will be influenced by the order or manner in which the choices are given. For this reason, wherever possible, the significant question was asked in more general terms such as: What about now?

There was also the possibility in this test to discover that the correct response to each item was the same. Asking for a reason for his reply would tend to keep S honest but further controls were built into the test as well. Five distracting questions were incorporated into the test to break the pattern of correct responses. The correct answer for each of these was different from those of the test proper. The first distracting question was asked during the introductory item after E had removed two red blocks. The second and third distractors were asked with the candy problem before Subtest 3 while the fourth and fifth distractors were asked when the two jars of water and the two jars of puffed wheat were first presented to S.

The Mathematical Achievement Test

The Seeing Through Arithmetic Test 1 (Hartung, et al, 1966) published by Scott Foresman and Company was admini-

stered and scored as per Teacher's Guide by the classroom teacher during the first two weeks of June, 1968. This is a group test designed to measure the class achievement in terms of the goals and content of the Seeing Through Arithmetic 1 program.

STAT 1 contains two parts. Part 1, consisting of twenty-four items, is designed to test the child's knowledge of, and his ability to apply, certain basic concepts of arithmetic. Part 2, consisting of thirty-six items, tests the child's knowledge of the basic facts that are presented in STA.

Items that relate to each content area of Seeing Through Arithmetic 1 are included in the test. The test items were validated in an experimental testing program that was conducted in the spring of 1965....

The items in STAT 1 involve only the mathematical ideas and the basic facts that are presented in Seeing Through Arithmetic 1. Thus, it can be expected that children who complete Seeing Through Arithmetic 1 will answer correctly a majority of the test items in STAT 1 (Teacher's Guide, STAT 1, p. 1).

III. ANALYSES USED

Since the conservation test used in the study was designed by the investigator, the relationship between the items was statistically checked by calculating the Pearson correlation coefficients. An item analysis of each of the subtests was carried out. The internal consistency of the test was assessed by computing the Kuder-

Richardson (KR 20) reliability coefficient.

The relationship between mathematics achievement, ability to conserve, intelligence, chronological age, sex, socio-economic status, and kindergarten/playschool attendance was found by examining the intercorrelations between all the variables. For those correlations that were found to be significant, partial correlations were calculated to assess the contribution of each of the variables towards the prediction of mathematics scores and conservation scores when the differential effects of the other variables were eliminated.

Differences between correlations were analyzed to find whether some of the conservation subtests are significantly superior in predicting mathematics scores.

The level for rejecting hypotheses was placed at 5 per cent. The computer programs supplied by the Division of Educational Research Services at the University of Alberta were used to carry out the above analyses.

CHAPTER IV

THE RESULTS OF THE INVESTIGATION

The results of the conservation test and the mathematics achievement test are summarized and analyzed in this chapter. The intercorrelations of age, sex, socio-economic status, kindergarten/playschool attendance, I.Q., conservation, and mathematics achievement are presented and analyzed as they relate to various topics under consideration.

In order to investigate certain factors which might affect the learning of mathematics, eighty-one grade one children (38 girls and 43 boys) were randomly selected to participate in the study. A conservation test (see Appendix A) consisting of five subtests was administered individually to each subject. The STAT 1 was given by the classroom teacher during the first two weeks of June, 1968. For the sample, the mean chronological age was 83.4 months with a standard deviation of 3.7 and the mean I.Q. was 102.4 with a standard deviation of 16.0. The distribution of I.Q. scores is illustrated in Figure 4. Although both extremes of the Blishen Occupational Class Scale were represented in the sample, the mean of 50.7 and the standard deviation of 8.9 indicate that the socio-economic status of the sample was normal for Canada. Fifty-eight of the subjects

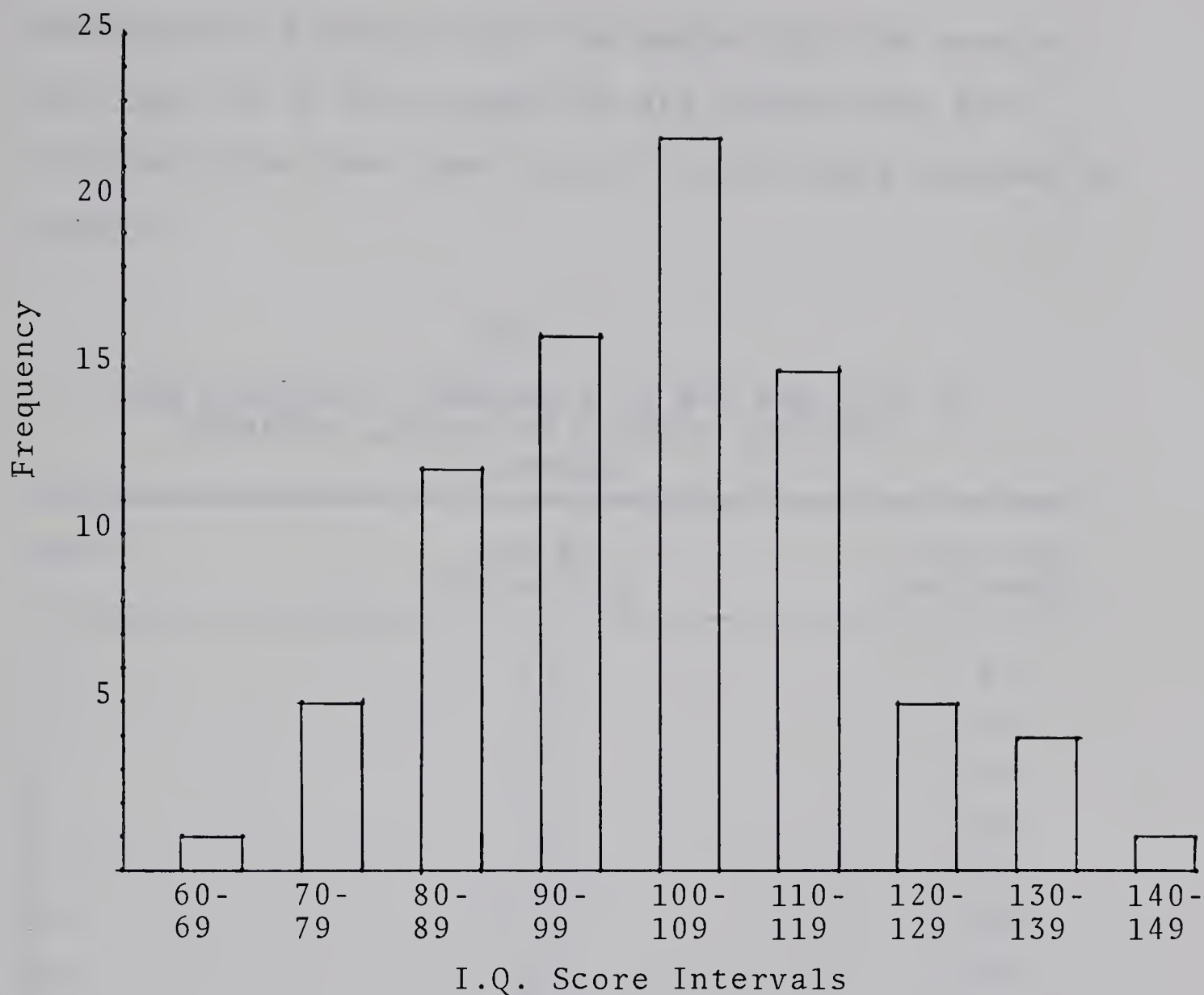


FIGURE 4

FREQUENCY DISTRIBUTION OF I.Q. SCORES
(N=81)

(71.6 per cent) had attended kindergarten or playschool.

I. CONSERVATION TEST

This ten-item test was designed to assess five areas of conservation. Table II shows the per cent of subjects who answered each item correctly. The item difficulty ranged from 51.9 to 76.5 thus indicating that no item was

inordinately difficult for the members of the sample. The items 4a to 5b on quantity and length were more difficult than the items 1a to 3b which were related to number.

TABLE II
ITEM DIFFICULTY EXPRESSED AS THE PER CENT OF
SUBJECTS GIVING THE CORRECT RESPONSE
(N=81)

Item	Number Successful	Per cent Successful
1a	60	74.1
1b	62	76.5
2a	61	75.3
2b	50	61.7
3a	62	76.5
3b	61	75.3
4a	42	51.9
4b	45	55.6
5a	43	53.1
5b	45	55.6

The distribution of conservation test scores is rather unusual. Although the proportion of subjects getting an item correct is nearly normal, the distribution of scores is not. As can be observed in Figure 5, a large proportion scored either very high or very low. The graph for the distribution of scores tends to a U-shape. Thus it would appear that a subject who was able to do several

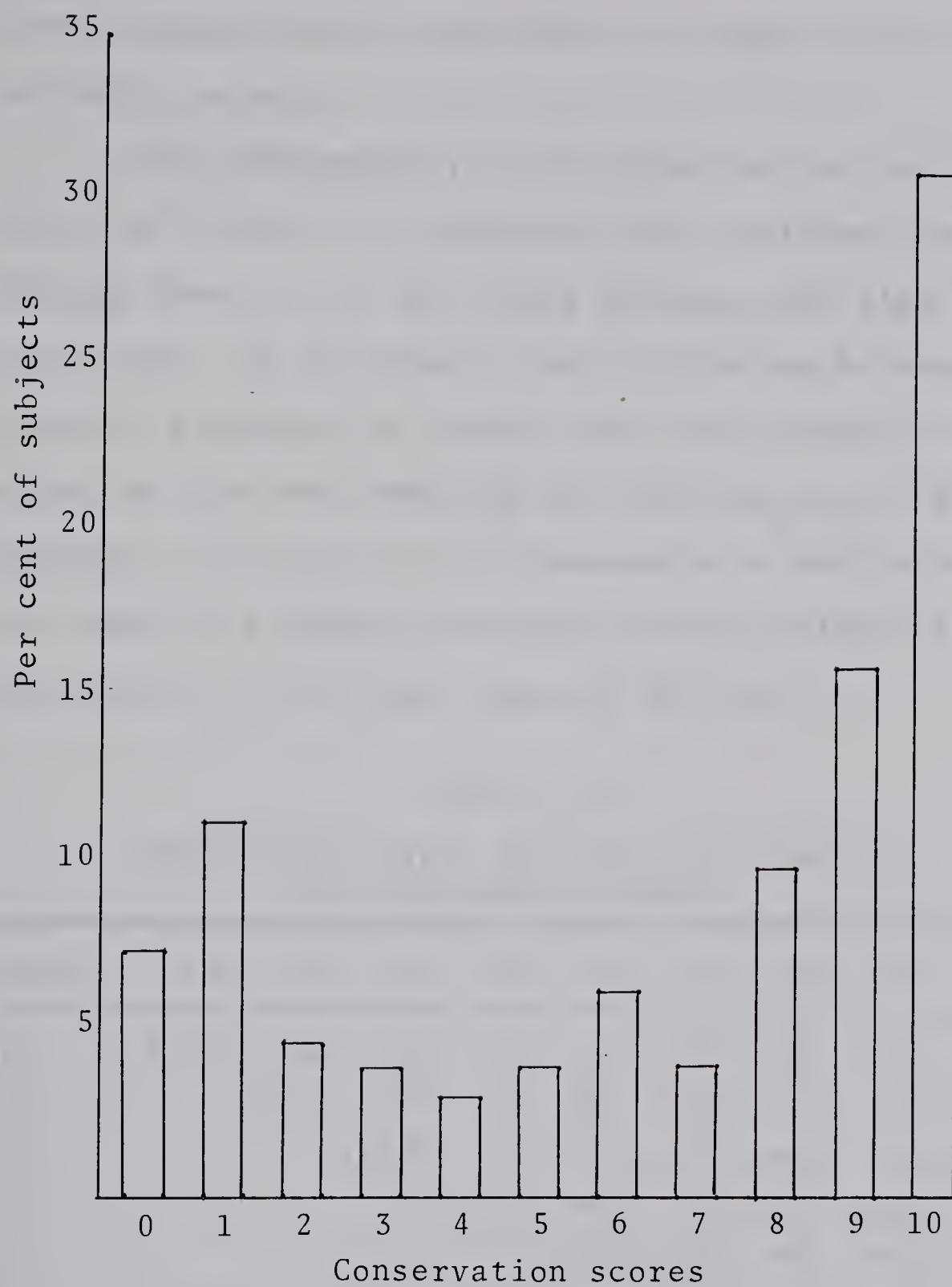


FIGURE 5
DISTRIBUTION OF CONSERVATION TEST SCORES
(N=81)

A measure of reliability for the conservation test was calculated using the Kuder-Richardson 20 formula. This formula provides an indication of the extent to which the distribution of the sample on each item parallels the distribution of the sample on the whole test, or more simply, the extent to which each item measures what the whole test measures. The reliability coefficient for this test was .848 indicating a high degree of internal consistency.

For some parts of the study it was necessary to know which subjects were successful in a specific sub-area of conservation. The criterion used to score the subtests individually was that a subject must perform correctly on both items of a subtest to be considered a conserver in that particular area. A score of one was assigned to the conservers and a score of zero was assigned to those who failed on one or both of the items. The per cent of subjects who were successful in each subtest when this criterion was used is given in Table IV. Again Subtests 4 and 5 were more difficult than the first three. While only 48.1 per cent of the subjects were able to conserve quantity and 48.1 per cent conserved length, 59.3 to 74.1 per cent were successful in the subtests which measured conservation of number.

TABLE IV

SUBTEST DIFFICULTY EXPRESSED AS THE PER CENT OF
SUBJECTS GIVING CORRECT RESPONSES ON BOTH ITEMS
OF A SUBTEST

Subtest	Number Successful	Per cent Successful
1	60	74.1
2	48	59.3
3	56	69.1
4	39	48.1
5	39	48.1

Results of Subtest 1

This subtest was designed to measure the subject's ability to realize that a small group of objects retains the same numerical characteristic despite transformation of the objects. For each item the subject was required to choose one of three possible answers. The distribution of these responses is given in Table V.

TABLE V

ANALYSIS OF RESPONSES FOR SUBTEST 1 EXPRESSED AS THE
PER CENT OF SUBJECTS CHOOSING EACH ALTERNATIVE

Item	More		Same
	<u>Long row</u>	<u>Short row</u>	
1a	24.7	1.2	74.1
	<u>Long row</u>	<u>Bunched</u>	
1b	21.1	2.4	76.5

In Item 1a, 24.7 per cent thought the longer row had more blocks, 1.2 per cent thought the shorter row had more blocks while 74.1 per cent insisted that the number of blocks in each row remained invariant. The distribution of responses for 1b was very similar.

When the conservers were asked to give a reason for their answers the majority argued that no blocks had been taken away, they were the same before, or they were just moved. The non-conservers, on the other hand, usually referred to the fact that one row was longer than the other or that some blocks were "sticking out" causing that row to have more blocks.

Results of Subtest 2

This subtest was designed to measure the subject's ability to realize that groups of objects greater than ten remain invariant under transformations. Table VI shows the per cent of subjects who chose each alternative.

TABLE VI

ANALYSIS OF RESPONSES FOR SUBTEST 2 EXPRESSED AS THE PER CENT OF SUBJECTS CHOOSING EACH ALTERNATIVE

Item	More		Same
	<u>Larger jar</u>	<u>Smaller jar</u>	
2a	9.9	14.8	75.3
2b	<u>Larger circle</u>	<u>Smaller circle</u>	61.7
	28.4	9.9	

Those who, for Item 2a, thought a higher level meant more beads responded that the small jar had more beads in it. Those who thought a larger container automatically meant a greater number of beads said the larger jar had more. Those who were not led astray by their perceptions maintained that both jars had the same number of beads. Their reasons were that no beads had been removed or that they were the same before. Only in a few cases did the subject say that the decrease in height was compensated for by the increase in diameter.

In Item 2b, the subject was not assisted in first making the discovery that both circles had the same number of objects. This probably explains why the number of correct responses is considerably less than for 2a. Of the non-conservers, 9.9 per cent chose the smaller circle as having more objects and 28.4 per cent chose the larger circle. The reason given by those who chose the smaller circle was that the spaces between the objects were smaller and thus there must be more objects. Those who said both circles had the same number of objects did so on the basis of matching the elements of the two sets such that a one-to-one correspondence was established.

Results of Subtest 3

This subtest was designed to measure the subject's ability to recognize that the total number of a given set

of objects remains invariant when the elements of the set are rearranged into different sized subsets. The distribution of responses is presented in Table VII. For both items, approximately three-quarters of the subjects gave the correct answer.

TABLE VII

ANALYSIS OF RESPONSES FOR SUBTEST 3 EXPRESSED AS THE PER CENT OF SUBJECTS CHOOSING EACH ALTERNATIVE

Item	More		Same
	<u>4+4 Arrangement</u>	<u>1+7 Arrangement</u>	
3a	16.1	7.4	76.5
	<u>Straight row of 9</u>	<u>Grouped (4+2+3)</u>	
3b	24.7	0	75.3

In Item 3a, at least two-thirds of the non-conservers thought two groups of four were more than a group of one and a group of seven. It is also interesting to note that in Item 3b no one thought that the three groups together (4+2+3) had more blocks than the straight row of nine blocks. Some of the typical reasons given for both items by the conservers were: "You didn't take any away", "You just moved them", or "They were the same before".

Results of Subtest 4

This subtest was designed to measure the subject's

ability to realize that a continuous quantity such as water or a semi-continuous quantity such as puffed wheat remains invariant when poured into different sized or shaped containers. The distribution of responses is recorded in Table VIII.

TABLE VIII

ANALYSIS OF RESPONSES FOR SUBTEST 4 EXPRESSED AS THE PER CENT OF SUBJECTS CHOOSING EACH ALTERNATIVE

Item	More		Same
	<u>One large jar</u>	<u>Two small jars</u>	
4a	7.4	40.7	51.9
4b	<u>Large jar</u>	<u>Small jar</u>	55.6
	2.4	42.0	

Just over one-half of the subjects answered Item 4a correctly. Of those who answered incorrectly, by far the majority thought that the two smaller jars together contained more water than the one larger jar. The fact that two jars were required to hold the water and the fact that the water level in these smaller jars was higher than in the larger jar convinced these subjects that the two smaller jars contained more water. The conservers, on the other hand, usually referred back to the state when both amounts were equal. They maintained that no water had been added or subtracted.

Item 4b was answered correctly by about 56 per cent of the subjects. In this item nearly all of the non-conservers thought the smaller jar full of puffed wheat contained more than the larger jar which was only about half full. Here again the comparison made by the non-conservers was on the basis of "fullness" or level while the conservers used the argument of identity. For both of the items several conservers used the argument of compensation, explaining that the increase in one dimension was compensated for by a decrease in some other dimension.

Results of Subtest 5

This subtest was designed to test the ability to realize that the length of an object remains invariant despite subdivision and transformations. The distribution of responses is recorded in Table IX.

TABLE IX

ANALYSIS OF RESPONSES FOR SUBTEST 5 EXPRESSED AS THE PER CENT OF SUBJECTS CHOOSING EACH ALTERNATIVE

Item	Longer		Same
	<u>Straight sidewalk</u>	<u>Cornered sidewalk</u>	
5a	45.7	1.2	53.1
	<u>Extended to left</u>	<u>Extended to right</u>	
5b	33.3	11.1	55.6

In Item 5a, about 46 per cent of those tested thought the straight "sidewalk" was longer than the "sidewalk built around a corner". Just over half of the subjects answered the problem correctly. Most of those who responded correctly argued that the "sidewalks" were of equal length originally and that nothing had been added or subtracted during the transformation.

The transformation in Item 5b consisted of sliding one stick about three-quarters of an inch to the left. About 33 per cent chose the stick extended to the left as being longer while only about 11 per cent chose the stick extended to the right. It would be presumptuous to attribute this difference to a left-hand dominance since a common reason expressed by the 33 per cent was that the stick was longer because it had been moved. The rationale expressed by the conservers was similar to that of Item 5a.

II. MATHEMATICAL ACHIEVEMENT TEST

The Seeing Through Arithmetic Test 1 published by Scott Foresman and Company was administered to each of the classes by the classroom teacher. Part 1, consisting of twenty-four items, is designed to test the child's knowledge of, and his ability to apply, certain basic concepts of mathematics. Part 2, consisting of thirty-six items, tests the child's knowledge of the basic addition and subtraction facts presented in STA 1. All of the subjects

had been using the STA materials during the nine-month period prior to the administration of the STAT 1.

The scores for Part 1, Concepts and Applications, ranged from a low of eight to a high of twenty-four. The distribution of the scores between these two extremes is shown in Figure 6. The scores tend to cluster around the upper intervals of the scale. A perfect score of twenty-four was attained by two subjects.

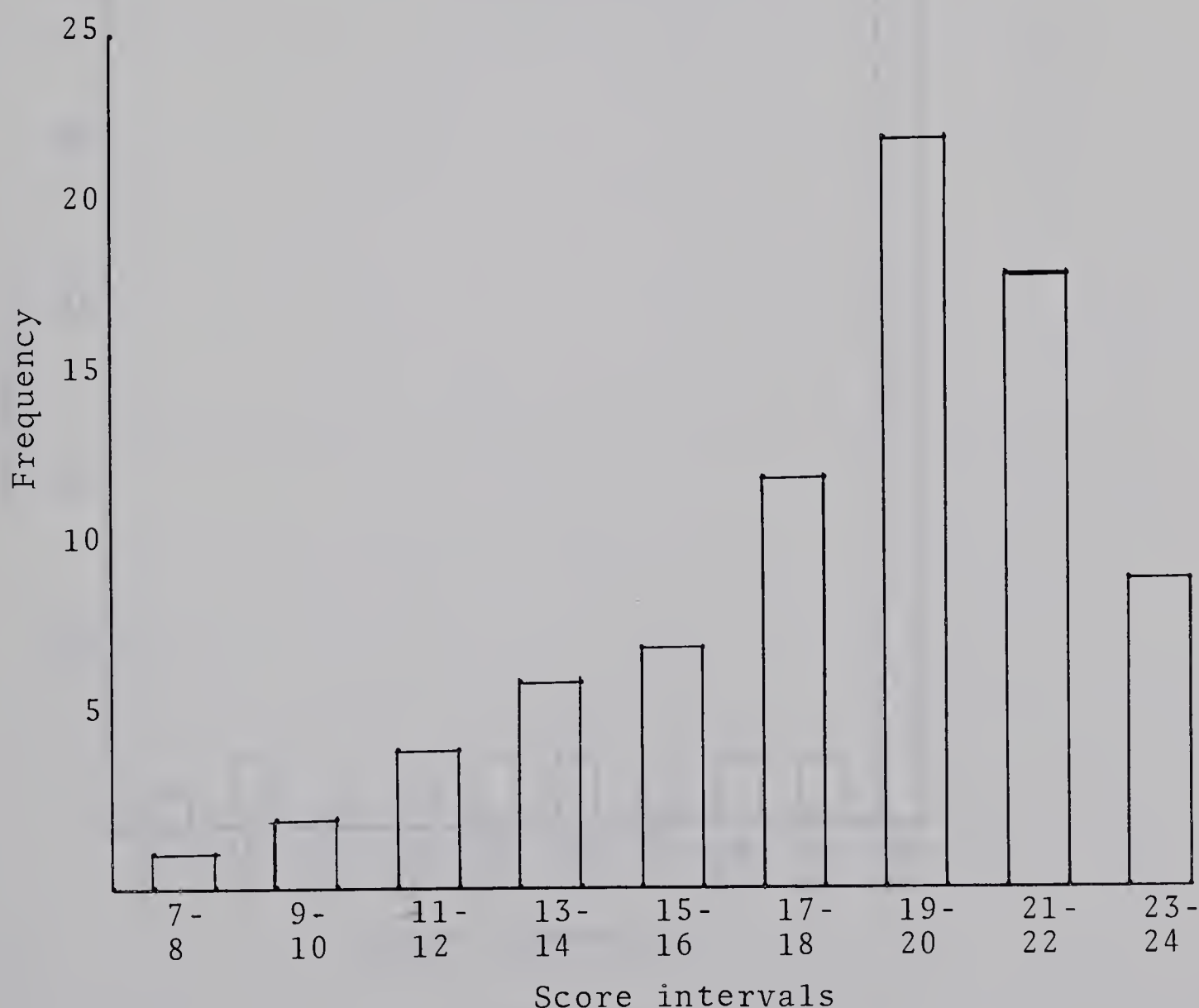


FIGURE 6

FREQUENCY DISTRIBUTION OF SCORES FOR PART 1 OF STAT 1
(N=81)

The scores for Part 2, Basic Facts, ranged from a low of eight to a high of thirty-six. As can be seen from Figure 7, the distribution is J-shaped. In fact, thirty-seven subjects had perfect scores on this part of the test. It is important to note that Part 2 requires automatic responses that are not necessarily based on understanding.

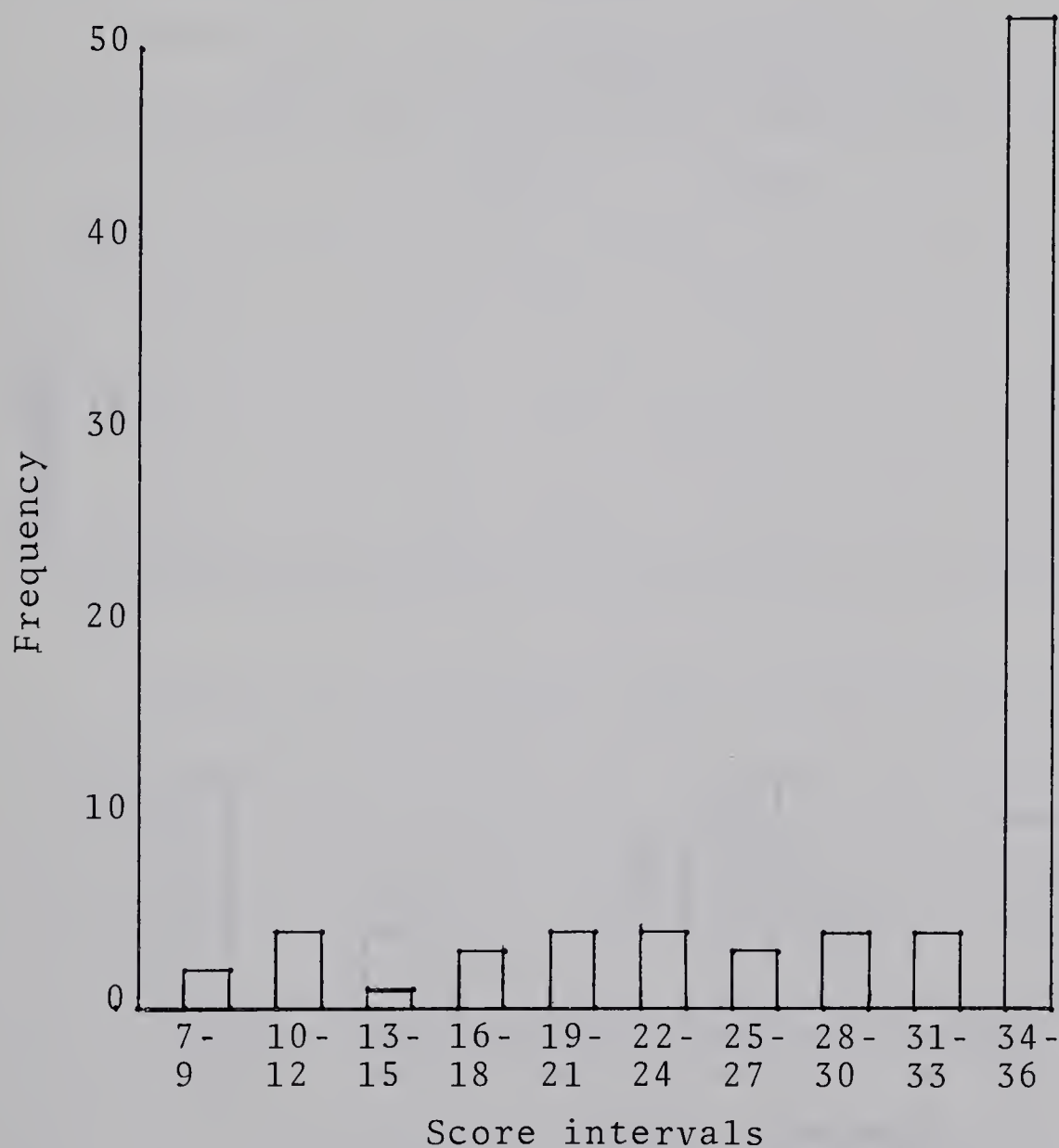


FIGURE 7

FREQUENCY DISTRIBUTION OF SCORES FOR PART 2 OF STAT 1
(N=81)

The distribution of the total STAT 1 scores is shown in Figure 8. The scores for the sample would indicate that the test was very easy for most of the subjects. The prediction of the authors, namely, that children who complete STA 1 will answer correctly a majority of the test items, is true for the greater portion of the sample. One subject earned a perfect score of sixty.

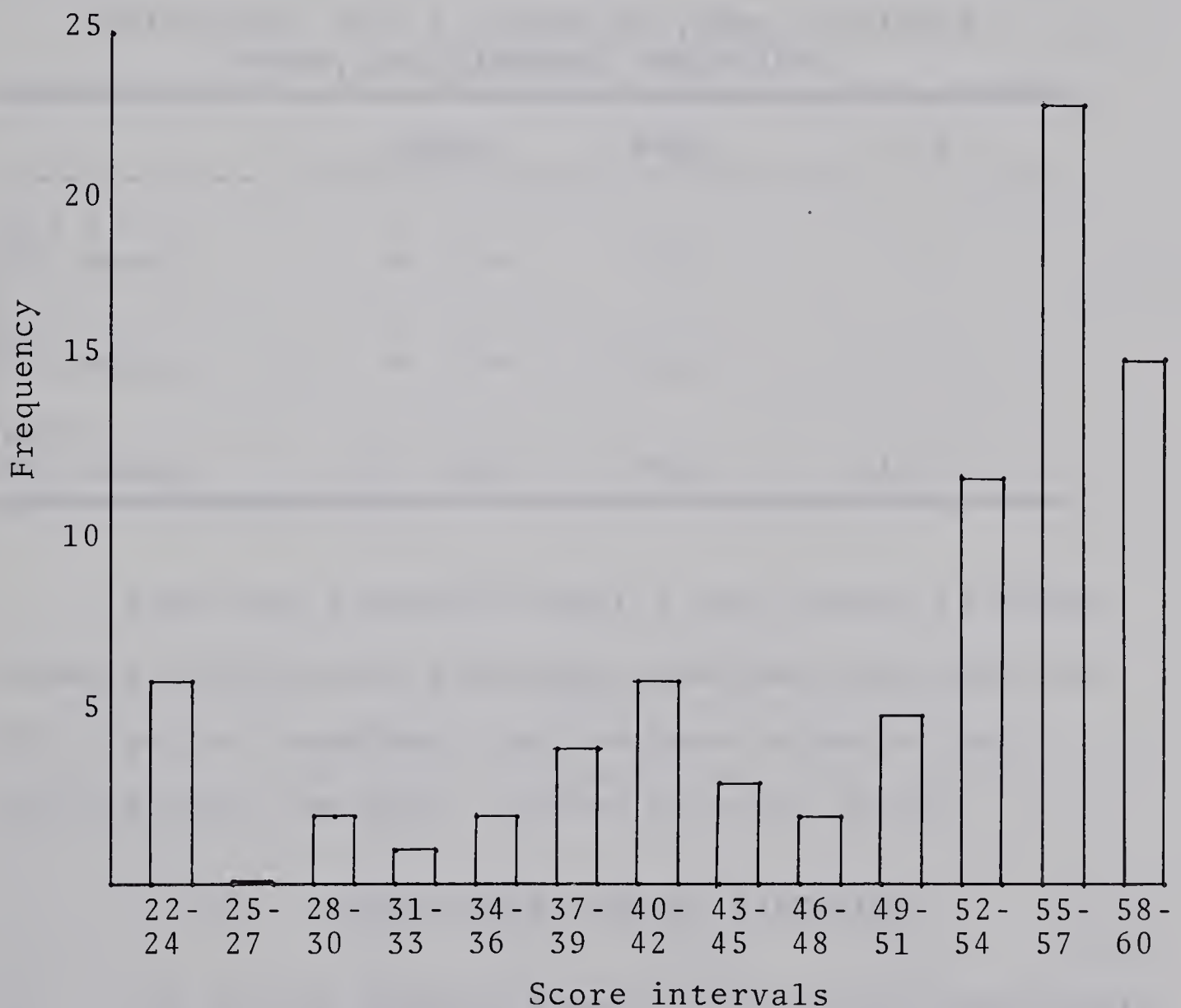


FIGURE 8

FREQUENCY DISTRIBUTION OF TOTAL SCORES FOR STAT 1
(N=81)

Further analyses of the STAT 1 results are given in Table X. The mean score for Part 1 (24 items) was 18.6; for Part 2 (36 items) it was 30.6; and for the total test (60 items) it was 49.1. The high means and the small standard deviations indicate that the scores were tightly clustered near the maximum values.

TABLE X
ANALYSIS OF STAT 1 SCORES IN TERMS OF RANGES,
MEANS, AND STANDARD DEVIATIONS

	Range	Mean	S.D.
Part 1 (24 items)	8 - 24	18.6	3.7
Part 2 (36 items)	8 - 36	30.6	8.2
Total (60 items)	22 - 60	49.1	10.7

Since the scores for Part 1 are closest to being normally distributed, subsequent analyses involving the STAT 1 scores from Part 1 may be more reliable than analyses which use Part 2 scores or total scores.

III. INTERCORRELATIONS OF VARIABLES

The Pearson product-moment correlation coefficients for all pairs of variables were computed using the Reg 200 computer program. Those correlations that are of significance to the study will be cited as they apply to the

various questions under consideration.

The Relation of Conservation to Variables Other Than
Mathematics Achievement

The correlation coefficients describing the relationship between conservation scores and the variables other than mathematics achievement are given in Table XI. A correlation of .219 is required to be significantly different from zero at the .05 level of significance with seventy-nine degrees of freedom on a two-tailed test. The only correlations greater than .219 were those involving I.Q.

Conservation and the various sub-areas of conservation do not correlate significantly with sex, age, socio-economic status, or kindergarten/playschool attendance. There is, however, a significant positive correlation between conservation and I.Q.

TABLE XI

CORRELATIONS OF CONSERVATION SCORES WITH AGE, SEX,
SOCIO-ECONOMIC STATUS, KINDERGARTEN/PLAYSCHOOL
ATTENDANCE, AND I.Q.

Conservation	Age	Sex	S.E.S.	Kinder-	I.Q.
Total Score	.092	.042	.037	.149	.452**
Subtests 1-3	.087	.079	.041	.151	.362**
Subtest 1	.093	.121	.093	.127	.303**
Subtest 2	.007	.127	.006	.091	.441**
Subtest 3	.121	.015	-.001	.172	.267*
Subtest 4	.103	-.035	-.062	.059	.399**
Subtest 5	.064	-.084	.090	.114	.476**

* Significant at the .05 level ($r \geq .219$)

** Significant at the .01 level ($r \geq .285$)

The relationship between the total conservation test scores and the I.Q. scores is illustrated in the scatter diagram of Figure 9. The correlation coefficient for this relationship was .452. Of the thirty-eight subjects who scored nine or ten on the conservation test, thirty-three (86.8 per cent) had an I.Q. score greater than one hundred. Of the forty-three subjects who scored less than nine on the conservation test, only thirteen (30.2 per cent) had an I.Q. score greater than one hundred. All of the fifteen subjects who scored zero or one on the conservation test had an I.Q. score of one hundred or less.

On the basis of these results it is with a high degree of confidence that I.Q. scores above one hundred can be predicted for subjects who had conservation scores of nine or ten. I.Q. scores below one hundred can be predicted for subjects who scored less than two on the conservation test.

The Relation of Mathematics Achievement to Variables Other Than Conservation

The correlation coefficients describing the relationship between mathematics achievement and the variables other than conservation are given in Table XII. The STAT scores do not correlate significantly with age, sex, socio-economic status, or kindergarten/playschool attendance.

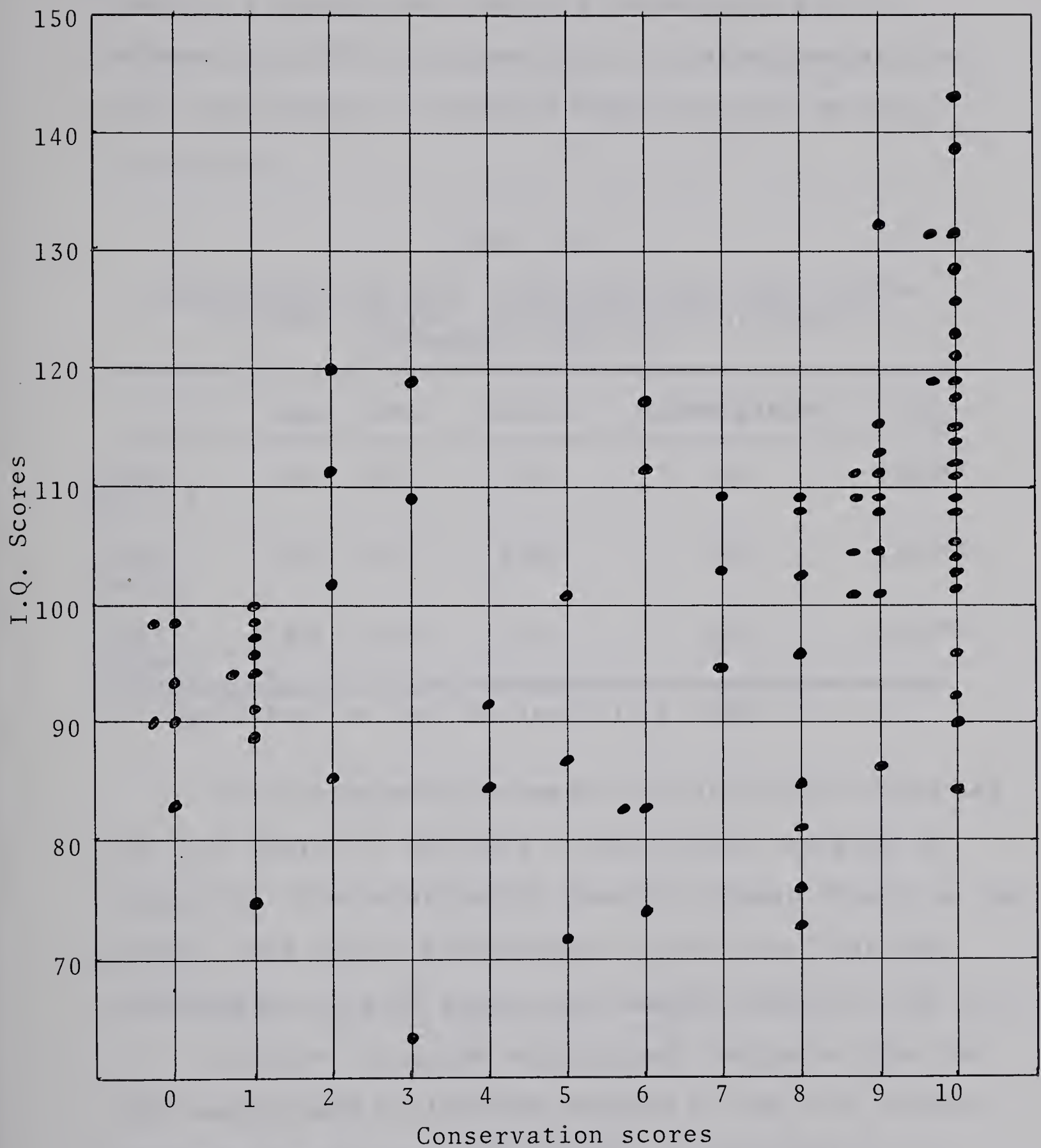


FIGURE 9

SCATTER DIAGRAM FOR THE TOTAL CONSERVATION TEST
 SCORES AND I.Q. SCORES
 ($r = .452$)

However, a significant positive correlation exists between the STAT scores and I.Q. A further analysis of this relationship is given in the section on Partial Correlations.

TABLE XII
CORRELATIONS OF STAT SCORES WITH AGE, SEX, SOCIO-ECONOMIC STATUS, KINDERGARTEN/PLAYSCHOOL ATTENDANCE AND I.Q.

	Age	Sex	S.E.S.	Kindergarten	I.Q.
STAT Part 1	.194	.057	.161	.034	.607**
STAT Part 2	.011	-.077	.096	.064	.543**
STAT Total	.074	-.038	.127	.060	.621**

** Significant at the .01 level ($r \geq .285$)

The relationship between the total STAT scores and the I.Q. scores is pictured in the scatter diagram of Figure 10. The relationship does not appear linear in the graph. This may be attributable to the fact that the distribution of STAT scores was sharply skewed to the left.

A closer analysis of the graph indicates that the STAT scores tend to increase rapidly as the I.Q. scores increase from the minimum to 110. Of the twenty-five subjects who had I.Q. scores greater than 110, only one subject earned a STAT score less than fifty-three out of a possible sixty. Thus a high STAT score can be

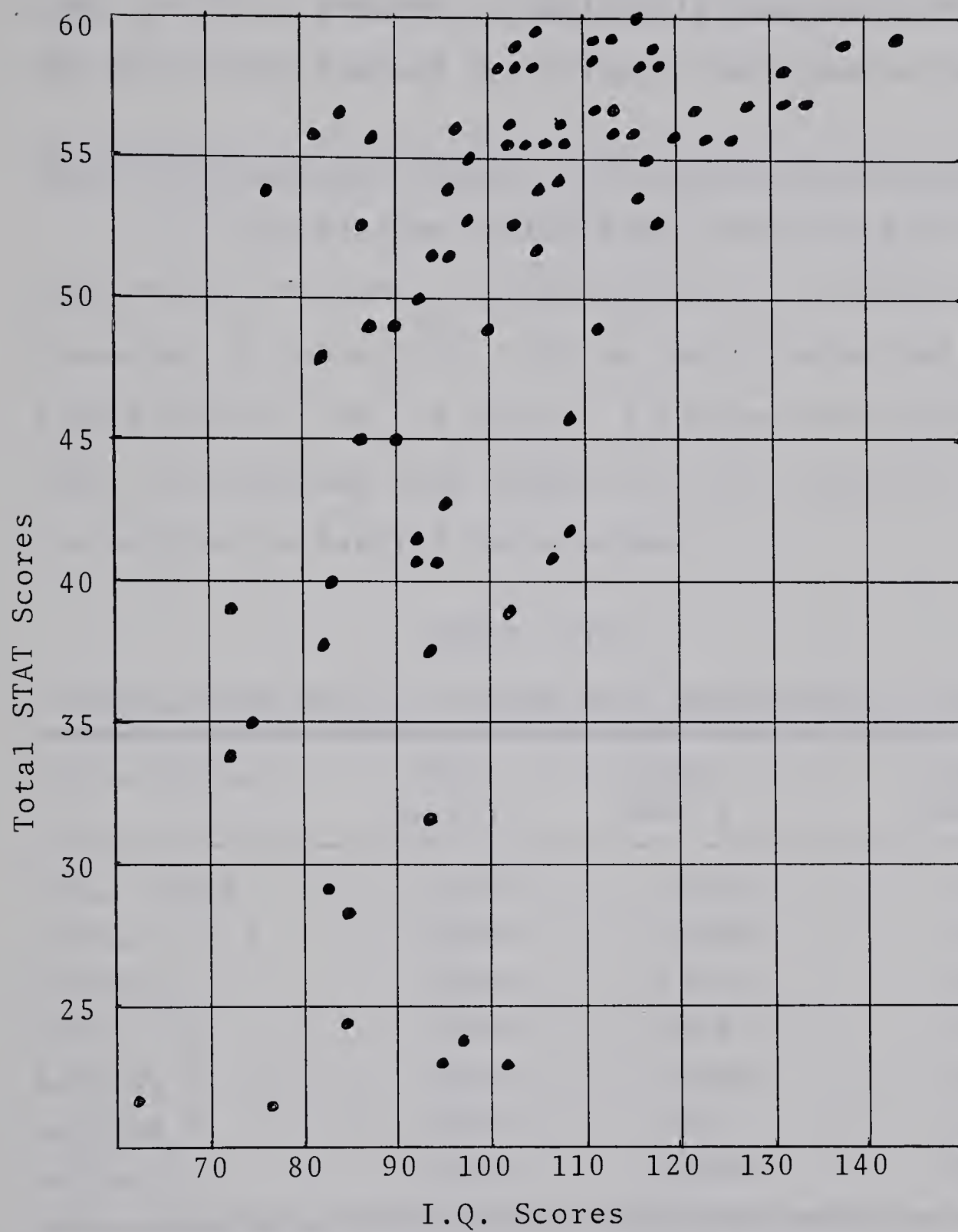


FIGURE 10

SCATTER DIAGRAM FOR THE TOTAL STAT SCORES AND
THE I.Q. SCORES
($r = .621$)

predicted for subjects whose I.Q. score is above 110.

That the STA 1 program is definitely manageable by those who have above average intelligence may also be concluded.

The Relation of Mathematics Achievement to Conservation

The correlation coefficients describing the relationship between mathematics achievement and conservation are presented in Table XIII. All of the correlations are significant at the .05 level. A further interpretation of these correlations with respect to I.Q. scores is given in the section on Partial Correlations.

TABLE XIII

CORRELATIONS OF STAT SCORES WITH CONSERVATION SCORES

Conservation	STAT Part 1	STAT Part 2	STAT Total
Total score	.414**	.354**	.412**
Subtests 1-3	.329**	.318**	.354**
Subtest 1	.316**	.297**	.334**
Subtest 2	.357**	.264*	.323**
Subtest 3	.244*	.289**	.303**
Subtest 4	.360**	.227*	.298**
Subtest 5	.393**	.396**	.438**

* Significant at the .05 level ($r \geq .219$)

** Significant at the .01 level ($r \geq .285$)

The relationship between the total conservation test scores and the total STAT scores is pictured in the scatter diagram of Figure 11. A survey of the graph indicates that of the thirty-eight subjects who scored nine or ten on the conservation test, thirty-two (84.2 per cent) earned a STAT score greater than fifty. Of the forty-three subjects who scored less than nine on the conservation test, only eighteen (41.9 per cent) received a STAT score greater than fifty. On the basis of these results it is with a high degree of confidence that STAT scores above fifty can be predicted for subjects who had conservation scores of nine or ten. In other words, when working with STA 1 materials the conservers seem to have a distinct advantage over the non-conservers.

IV. DIFFERENCES BETWEEN CORRELATIONS

That each of the conservation subtests is significantly correlated with mathematics achievement can be observed in Table XIII. However, a further purpose of this study was to determine whether some types of conservation are more closely related to mathematics achievement than some other types. One might expect the subtests on number conservation to correlate more highly with the STAT scores than the subtests on conservation of quantity or length because only two questions in the STAT 1

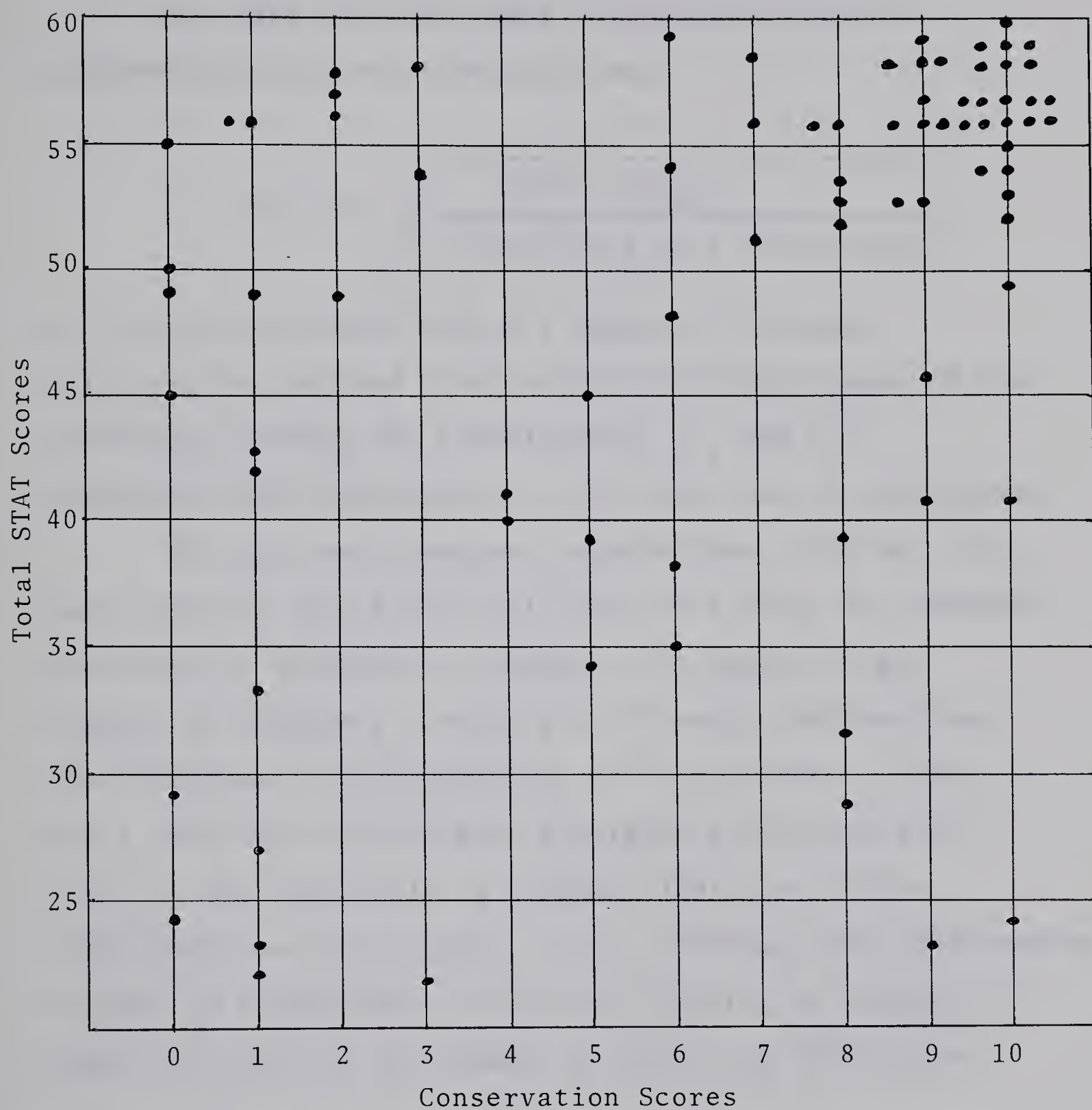


FIGURE 11

SCATTER DIAGRAM FOR THE TOTAL CONSERVATION TEST SCORES
AND THE TOTAL STAT SCORES
($r = .412$)

seem directly related to the latter two subtests.

The test statistic used to determine whether differences in correlation exist was

$$t = (r_{12} - r_{13}) \sqrt{\frac{(N-3)(1+r_{23})}{2(1-r_{23}^2 - r_{12}^2 - r_{13}^2 + 2r_{23}r_{12}r_{13})}},$$

a $\underline{t}$ ratio distributed with $N-3$ degrees of freedom.

Hotelling has devised this test of the significance of the difference between two correlations, r_{12} and r_{13} , calculated from observations on the same set of individuals.

The two most divergent correlations, .396 and .227, were selected (see Table XIII) and the $\underline{t}$ ratio was computed according to Hotelling's formula. For seventy-eight degrees of freedom a $\underline{t}$ ratio ≥ 1.991 would indicate that the differences in correlations are significant. Since the $\underline{t}$ ratio for the two most divergent correlations was 1.752 it was reasonable to conclude that none of the differences was significant. Thus, although each conservation subtest is correlated with the STAT scores, no subtest is superior to any of the others in predicting STAT scores.

V. PARTIAL CORRELATIONS

All or part of the correlation between two variables, X_1 and X_2 , may result because both are correlated with a third variable, X_3 . The part of the correlation which

remains when the effect of the third variable is eliminated or removed is called a partial correlation.

The formula for calculating the partial correlation coefficient to eliminate the effects of a third variable is

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1-r_{13}^2)(1-r_{23}^2)}}.$$

A t test may be used to test whether a partial correlation coefficient is significantly different from zero. The required t is

$$t = \frac{r_{12.3}}{\sqrt{(1-r_{12.3}^2) / (N-3)}}.$$

This may be referred to a table of t with N-3 degrees of freedom. (Ferguson, 1959, p. 291)

Partial correlations and t ratios were computed to assess the contribution of each of the variables, conservation and intelligence, towards the prediction of mathematics achievement scores in the presence of the other variable. A computer program for IBM/67 using Iverson's APL notation was used to make the calculations. This statistical procedure is essentially the same as the multiple linear regression technique. An example is given to show that both techniques produce the same results.

Suppose it is necessary to assess the contribution

of conservation scores towards the prediction of STAT scores in the presence of I.Q. The variables may be identified as follows:

X_1 = Total STAT scores (Criterion)

X_2 = Conservation scores (Variable 2)

X_3 = I.Q. scores (Variable 3)

Using the partial correlation technique as outlined above produces the following results.

Given: $r_{12} = .412$, $r_{23} = .452$, $r_{13} = .621$

Partial correlation coefficient = .188

t ratio = 1.692

t value significant at the .05 level with seventy-eight degrees of freedom = 1.991

Conclusion: The contribution of conservation scores towards the prediction of STAT scores in the presence of I.Q. is not significant.

For the multiple linear regression technique it is necessary to set up a full model and a restricted model. These two models are compared and an F ratio is computed which may be referred to a table of F with the appropriate degrees of freedom.

The following models are appropriate for the problem of assessing the contribution of conservation scores towards predicting STAT scores in the presence of I.Q.

Full Model: $X_1 = a_0U + a_1X_2 + a_2X_3 + e_1$

Restricted Model: $X_1 = a_3U + a_4X_3 + e_2$

The results of the computation may be summarized as follows:

F ratio = 2.864

F value significant at the .05 level with one degree of freedom in the numerator and seventy-eight degrees of freedom in the denominator = 3.96.

Conclusion: The contribution of conservation scores towards the prediction of STAT scores in the presence of I.Q. is not significant.

It should also be noted that a direct algebraic relationship exists between the F ratio with one degree of freedom in the numerator and the t ratio if the degrees of freedom in the denominator for F and the degrees of freedom for t are equal. This relationship may be expressed as $\text{F ratio} = (\text{t ratio})^2$. A direct comparison of the F ratio and the t ratio from the example above shows that the two values satisfy the equation, which proves in a more direct way that the partial correlation technique and the multiple linear regression technique lead to the same conclusions.

Partial Correlation of Conservation Scores with STAT Scores

The parts of the correlation between conservation scores and STAT scores that remain when the differential effects of I.Q. are eliminated are given in Table XIV.

None of the partial correlations is significant at the .05 level. Therefore, knowing the conservation scores does not significantly improve the prediction of STAT scores made on the basis of I.Q. scores.

TABLE XIV

PARTIAL CORRELATIONS OF CONSERVATION SCORES WITH
STAT SCORES WHEN THE EFFECT OF I.Q. IS ELIMINATED

Criterion (STAT scores)	Predictor (Conservation)	Partial Correlation	<u>t</u> ratio
Total	Total	.188	1.692
Part 1	Total	.198	1.781
Part 2	Total	.145	1.293
Part 2	Subtests 1-3	.156	1.391
Total	Subtests 1-3	.178	1.595
Total	Subtest 1	.195	1.758
Total	Subtest 2	.070	.616
Total	Subtest 3	.182	1.635
Total	Subtest 4	.070	.617
Total	Subtest 5	.207	1.870

Note: A t of 1.991 indicates significance at the .05 level.

Partial Correlations of I.Q. Scores with STAT Scores

In order to ascertain whether I.Q. is correlated with mathematics achievement when the differential effects of conservation are eliminated, partial correlations between I.Q. scores and STAT scores were calculated. These, together with the respective t ratios, are presented in

Table XV. In every case the partial correlation is significant. Therefore knowing the I.Q. scores does improve the prediction of STAT scores made on the basis of conservation scores.

TABLE XV

PARTIAL CORRELATIONS OF I.Q. SCORES WITH STAT SCORES
WHEN THE EFFECT OF CONSERVATION IS ELIMINATED

Criterion (STAT scores)	Predictor	Partial Correlation	<u>t</u> ratio
Total	I.Q.	.535	5.593**
Part 1	I.Q.	.517	5.333**
Part 2	I.Q.	.459	4.565**

** Significant at the .01 level ($t \geq 2.640$)

VI. SUMMARY OF RESULTS

The principal findings in relation to the hypotheses are briefly summarized.

Hypothesis 1: There is no significant relationship between conservation and the variables of chronological age, sex, socio-economic status, or kindergarten/playschool attendance.

This hypothesis was accepted. None of the respective correlations was significant.

Hypothesis 2: There is no significant relationship between conservation and intelligence.

This hypothesis was rejected. Those with higher I.Q. scores tended to get higher conservation scores.

Hypothesis 3: There is no significant relationship between mathematics achievement and the variables of chronological age, sex, socio-economic status, or kindergarten/playschool attendance.

This hypothesis was accepted. None of the respective correlations was significant.

Hypothesis 4: There is no significant relationship between mathematics achievement and intelligence.

This hypothesis was rejected. Those with higher I.Q. scores tended to get higher STAT scores.

Hypothesis 5: There is no significant relationship between mathematics achievement and intelligence when the differential effects of conservation are eliminated.

This hypothesis was rejected. The correlations that remained after eliminating the effects of conservation were still significant.

Hypothesis 6: There is no significant relationship between mathematics achievement and conservation.

This hypothesis was rejected. Those with high conservation scores tended to get high STAT scores.

Hypothesis 7: There is no significant relationship between mathematics achievement and conservation when the differential effects of intelligence are eliminated.

This hypothesis was accepted. The correlations that remained after eliminating the effects of I.Q. were not significant.

Hypothesis 8: The correlations of different kinds of conservation with mathematics achievement are not significantly different.

This hypothesis was accepted. The correlations did not differ significantly.

CHAPTER V

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND SUGGESTIONS FOR FURTHER RESEARCH

The purpose of this study was to investigate the effect of certain factors upon achievement in first grade mathematics. Attention was focused particularly on the concept of conservation and its relationship to mathematics achievement.

A sample of eighty-one children was selected at random from eight grade one classrooms within the public school system of Edmonton, Alberta, Canada. The sample consisted of thirty-eight girls and forty-three boys. Of these, fifty-eight pupils (71.6 per cent) had attended kindergarten or playschool. The ages of these children ranged from six years five months to seven years ten months. The intelligence quotients, taken from available school records of Detroit Beginners First Grade Intelligence Tests ranged from sixty-three to one hundred forty-three. For the sample, the ratings of socio-economic status, as measured by the Blishen Occupational Class Scale, ranged from 38.8 to 78.8. These children appeared to represent typical grade one students in urban school systems.

The development of the concept of conservation was examined by means of a test constructed by the investigator. This test consisted of five subtests, each designed to

measure a specific type of conservation. The first subtest measured conservation of number less than ten; the second measured conservation of number greater than ten; the third measured conservation of number in an additive rearrangement; the fourth measured conservation of quantity; and the fifth measured conservation of length.

The conservation test was administered to each subject in an individual testing situation during the period of May 10th to 22nd, 1968. The mathematics achievement test (Seeing Through Arithmetic Test 1) was administered to each class by the classroom teacher during the first two weeks of June, 1968. All classes had been using the STA materials during the school year.

The data were analyzed and the hypotheses were tested with the help of computer programs supplied by the Division of Educational Research Services, Faculty of Education, at the University of Alberta.

I. CONCLUSIONS

On the basis of testing the hypotheses, the following conclusions may be drawn.

(1) No significant relationship exists between conservation and the variables of chronological age, sex, socio-economic status, or kindergarten/playschool attendance. It would appear that the common experiences during the first year at school overcame any differences in the understanding

of conservation that may have existed due to pre-school experiences. Since the age range of the subjects in this study was restricted, the non-significant relationship between age and conservation is not generalizable to a population whose age span is greater than one year.

(2) A significant positive relationship exists between conservation (number, quantity, and length) and intelligence. This finding is in accord with those of Elkind (1961), Goodnow and Bethon (1966), Feigenbaum (1963), and Pelletier (1966).

(3) With first grade children no significant relationship exists between mathematics achievement and the variables of age, sex, socio-economic status, or kindergarten/playschool attendance. When Brace (1963) studied pre-school children's concepts of number he found no significant difference between boys and girls. However, he found both socio-economic status and chronological age to be significant predictors of the child's understanding of number. A possible explanation for this discrepancy in the results of these two studies is that the educational system may not be flexible enough to allow the older children and those from higher socio-economic homes to advance academically at a pace that is commensurate with their abilities.

(4) A highly significant relationship exists between mathematics achievement and intelligence both before

and after the effects of conservation are eliminated. The results of this study indicate that subjects with above average intelligence generally score high on the STAT 1. That mathematics achievement scores are positively correlated with intelligence scores is supported by many previous studies.

Removing the differential effects of conservation from the correlation of mathematics achievement with intelligence has not been reported previously. In this study it was found that the relationship between mathematics achievement and intelligence is significant even when conservation scores are held constant. This finding seems to indicate that intelligence tests measure a more inclusive set of cognitive abilities than conservation tests. For this reason it may be assumed that I.Q. scores are superior to conservation scores in predicting mathematics achievement.

(5) A significant relationship exists between mathematics achievement and conservation but knowledge of the ability to conserve does not contribute significantly more to prediction of mathematics scores if intelligence scores are known. Generally those who did well on the conservation test were also very successful on the mathematics achievement test. The results of this study indicate that for pupils who scored nine or ten on the conservation test, STAT scores above fifty can be predicted with a high degree of confidence. However, there were

several pupils who were highly successful in mathematics but who failed to conserve number, quantity, or length.

The results obtained when the differential effects of intelligence were eliminated were similar to those obtained by Overholt (1964) and Steffe (1966). The ability to conserve may be used as a predictor of the child's mathematical achievement but for the entire population the level of prediction would not be as reliable as that provided by intelligence. An examination of Figure 11 indicates, however, that pupils who were conservers of number, quantity, and length were, with few exceptions, also high scorers on the mathematics achievement test. Thus, being able to conserve appears to be of particular benefit to students of first grade mathematics.

(6) The correlations of different kinds of conservation with mathematics achievement are not significantly different. In predicting achievement even the subtests on quantity or length which seemed unrelated to most of the items in the STAT 1 were as reliable as the subtests related specifically to number. One probable explanation for this result is that the conservation subtests were measuring some general type of ability or level of development. Another explanation may be that the concept of conservation cannot be subdivided in any meaningful way. Conservation may be a comprehensive kind of concept which is simply extended in some fixed pattern

to include different kinds of ideas. A third explanation may be that the experiences of the subjects were so similar that no major digression from the pattern was likely. Success on the various sub-areas of conservation would therefore be closely related to intelligence and one could expect every subtest to predict achievement equally well.

During the course of completing the study certain results not directly related to the hypotheses were obtained as well.

(7) Approximately 75 per cent of the subjects, whose average age was about seven years, were successful in the conservation of number items. This agrees with Piaget's findings.

(8) The conservation problems involving quantity and length were solved correctly by approximately 50 per cent of the sample. These concepts are either more difficult than those involving number, they do not receive as much time and attention in the instructional program, or they naturally develop later in the systematic cognitive growth of the child. The latter of these explanations is the one suggested by Piaget's theory.

(9) Piaget emphasizes the importance of reversibility for making correct judgments in problem situations such as those presented in the conservation test. Although no thorough analysis was made of the reasons expressed by

the conservers, approximately three-quarters of the explanations indicated the use of identity and the other one-quarter indicated the use of either reversibility or compensation in solving the conservation problems.

II. IMPLICATIONS

Some suggestions for the planning and teaching of primary mathematics courses arise from the findings of this study.

(1) Since approximately 50 per cent of the subjects of this study failed to understand some of the basic concepts about quantity and length even at the end of grade one, it is suggested that teachers guard against formally introducing measurement to entire grade one or two classes without first assessing each child's level of development. The research of Piaget, Dienes, and Biggs indicates that activity, inquiry, experimentation, and discovery on the part of the child will encourage the development of basic measurement concepts which are necessary if future measuring activities are to be meaningful.

(2) Some subjects who did poorly on the conservation test scored high on the STAT. This suggests that the evaluative instruments such as STAT used by teachers may not, for certain children, differentiate between meaningful learning and rote learning. As a result, some children who really do not understand the mathematical ideas involved

are not detected and may not get the additional assistance that is required if subsequent concepts are to be developed meaningfully. Simple tests, similar to those used by Piaget, provide a quick analysis of a child's level of understanding and may well be used by teachers as a basis for planning mathematics lessons geared to the needs of the individual.

(3) Approximately 25 per cent of the subjects failed at the end of grade one to understand some of the basic concepts of number as tested by Subtests 1, 2, and 3 of the conservation test. According to Piaget, this may be a source of confusion to mathematics students. Teachers should be aware of this possibility in their own classrooms and should consider providing alternative experiences for such students so that the basic concepts will be developed before the prescribed course work is begun.

(4) The results of this study indicate that I.Q. scores are significantly correlated with mathematics scores. This finding suggests that intelligence tests such as the Detroit Beginners First Grade Intelligence Test can provide a criterion that may be used at the outset to group children for mathematics instruction.

(5) Differences in mathematical ability due to kindergarten/playschool attendance, chronological age, or socio-economic status that may have existed at the beginning

of the school year seem to be decreased during the course of the first school year. Effective teaching, however, is known to increase the differences between children. It is suggested that educators continue to study the possibilities of permitting continuous development of the individual. The educational system should allow those entrants who are further advanced due to their pre-school experiences to begin at their own level and to proceed at their own rate.

(6) Since cognitive development is related to language development, teachers should emphasize the use of precise terminology. The non-conservers in this study failed to realize the limitations of global comparison words such as 'bigger', 'higher', and 'longer'. A deliberate effort should be made to provide experiences which point out differences between such terms as 'longer row' and 'more blocks', 'larger cup' and 'more drinks', or 'bigger circle' and 'more children'.

(7) Since a much greater proportion of the conservers, subjects scoring nine or ten on the conservation test, scored above fifty on the STAT 1, it is suggested that a conservation test can be used as an indicator of the prerequisites that are necessary for mathematics learning as well as a predictor of mathematics achievement. Those who possess the concept of invariance appear to have a distinct advantage in the mathematics program over non-

conservers. Every effort should be made to ensure that children possess certain concepts of conservation before proceeding with the mathematics instruction prescribed by most textbooks.

III. SUGGESTIONS FOR FURTHER RESEARCH

Some suggestions for further research arising out of this study are presented.

(1) Intelligence and conservation ability were found to be significant predictors of the child's achievement in mathematics. Further research into the interaction effects of intelligence and the ability to conserve might prove useful. Such research would seek to analyze to a greater extent than the partial correlation technique is able to do the three-way interrelationship of conservation, intelligence, and mathematics achievement.

(2) Steffe (1966) mentions that memorization can take place before a child is able to overrule his perceptions. Future research into the factors which influence mathematics achievement might employ a criterion test which is based on understanding and which is administered and scored by someone other than the classroom teacher.

(3) This study has concluded that the five areas of conservation which were identified and tested do not differ significantly in their relationship to mathematics achievement. Future research might include further types of

conservation such as substance, distance, area, volume, and different kinds of quantity. The relationship of the various types of conservation to mathematics achievement in general and to individual topics specifically could be investigated.

(4) This study did not determine when each subject had acquired the various concepts of conservation. Future research of a longitudinal nature might establish when each child attains a specific concept and how this acquisition affects subsequent mathematics achievement. Such a longitudinal study would also provide data which would help answer the question of whether the concepts are acquired in the same order by every child.

(5) Smedslund (1961) and Towler (1967) were successful in inducing conservation in an experimental setting. Valuable information about the importance of conservation to mathematical achievement may be obtained by teaching conservation to non-conservers and then comparing their improvement in mathematical ability with that of a control group.

(6) A superficial analysis of the explanations given by the conservers was carried out in this study. The methods or logic that children use to solve conservation problems provide worthwhile material for future research. A thorough study of identity, reversibility, and compensation applied to cognitive development may give further insight into the

process of learning.

(7) In this study, the concepts involving quantity and length were more difficult for the sample as a whole than concepts about number. The reasons why some concepts develop later than others are not clear and provide interesting possibilities for further research.

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A P P E N D I C E S

APPENDIX A

CONSERVATION TEST

Introductory Item

Apparatus: 7 blue and 7 red wooden blocks each
3/4 inch cube.

E places 7 blue blocks in a straight row with approximately one-half inch spacing between blocks.

"Now, you put just as many red blocks here (E indicates a line parallel to blue blocks) as there are blue ones."

After S has completed the task E asks, "Are there more blue blocks, more red blocks, or the same number of blocks in each row?" (Before proceeding, S must agree they are equal.)

E removes the fourth and fifth red blocks.

"What about now?"*

E replaces the two red blocks.

"What about now?"

* If S fails to understand the problem whenever this question is asked, the complete question with the three alternatives is repeated.

Subtest 1: Conservation of Number Less Than Ten

Item 1a. Apparatus: Same as for the introductory item.

The final arrangement of blocks for the introductory item is used as a starting point for Subtest 1.

E spreads the red blocks to approximately one-inch spacings.

"What about now?"

"Why?"*

Item 1b. Apparatus: Same as for 1a.

E moves the red blocks back to one-half inch spacings.

"What about now?" (S must agree to equivalence before proceeding.)

E moves red blocks into a close bunch.

"What about now?"

"Why?"

Subtest 2: Conservation of Number Greater Than Ten

Item 2a. Apparatus: 30 large wooden beads, 2 identical small jars, 1 larger jar.

"Take a bead in each hand and drop them into these jars like this until all the beads are gone." (E demonstrates how the beads are dropped into the 2 smaller jars.)

* For some items this question may be changed to "How do you know?"

"Did you drop more beads into this jar, more beads into this jar, or are there the same number of beads in each?" (S must agree to equivalence before proceeding.)

E pours the beads from one jar into the larger jar.

"What about now?"

"Why?"

Item 2b. Apparatus: 2 12-inch square sheets of gray paper on which 12 yellow one-inch square pieces of paper and 12 blue one-inch square pieces of paper are arranged in two concentric circles having diameters of 4 inches and 7 1/2 inches respectively.

"What would you call these things?" (E points to several of the small "squares".)

"Are there more yellow (E uses S's own term), more blue _____, or the same number of _____ each?"

"Why?"

Distractor

Apparatus: one saucer with 3 candies and one saucer with 4 candies.

"Are there more candies in one saucer than in the other?"

"Would you like to take one of these (E points to saucer with 3 candies) and eat it?"

"What about now? Are there more candies in here, more candies in here, or the same number in each?"

"Why?"

Subtest 3: Conservation of Number in an Additive
Rearrangement

Item 3a. Apparatus: 2 sheets of blue paper having dimensions of 9" x 12", 16 Ritz crackers.

"Sometimes when you are home you like to have a lunch between meals. Suppose your mother says you may have 4 of these crackers for a morning lunch and 4 of them for an afternoon lunch. (E places 8 crackers on one sheet of paper in 2 groups of 4.) But the next day you want some lunches, too, and your mother says you may have the same thing: 4 crackers in the morning and 4 crackers in the afternoon." (E places 8 crackers on the second sheet of paper in 2 groups of 4.)

"Are there more crackers on this sheet, more crackers on this sheet, or the same number on both?" (Before proceeding, S must agree to equivalence.)

"But suppose that on this second day you are not very hungry in the morning so you eat only one cracker and save all the others for the afternoon." (E transfers 3 crackers from one group of 4 to the other group of 4, leaving a final arrangement of a group of 1 and a group of 7.)

"What about now? Are there more crackers on this sheet, more on this sheet, or the same number on each?"

"Why?"

Subtest 4: Conservation of Quantity

Item 4a. Apparatus: 2 identical jars with unequal amounts of colored water, 2 smaller identical jars.

E places the 2 larger jars with water in front of S.

"Is there the same amount of water in these 2 jars?"

E equalizes the contents to S's satisfaction.

"Are they the same now?" (S must agree that both jars have the same amount of water before proceeding.)

E pours the water from one of the jars into 2 smaller jars.

"Now, is there more water in this jar (E points to larger jar), more water in these 2 jars together (E points to the 2 smaller jars), or is there the same amount of water in both?"

"Why?"

Item 4b. Apparatus: 2 identical larger jars with unequal amounts of puffed wheat, a smaller jar.

E places the 2 larger jars with puffed wheat in front of S.

"Is there the same amount of puffed wheat in these 2 jars?"

E equalizes the contents to S's satisfaction.

"Are they the same now?" (S must agree that both jars have the same amount of puffed wheat before proceeding.)

E pours the puffed wheat from one of the jars into the smaller jar.

"Now, is there more puffed wheat in this jar (E points to larger jar), more in this jar (E points to smaller jar), or the same amount in both jars?"

"Why?"

Subtest 5: Conservation of Length

Item 5a. Apparatus: 2 identical strips of gray construction paper having dimensions of 6" x 1/2". One strip is cut in two with a 45° angle cut.

E places the 2 strips of paper in front of S. They are parallel, coterminous, and about 1/2 inch apart.

"Let's pretend these are 2 sidewalks. Is this sidewalk longer, is this sidewalk longer, or are they both the same length?" (Before proceeding, S must agree they are equal in length.)

"But what if I want to build this sidewalk around a corner like this? (E makes the necessary transformation with one part of the "cut" sidewalk.) Now, is this sidewalk longer, is this sidewalk longer, or are they both the same length?"

"Why?"

Item 5b. Apparatus: 2 identical sticks 6 inches in length.

E gives the 2 sticks to S and asks, "Are these 2 sticks the same length?"

E assists S in placing the sticks parallel and coterminous. (Before proceeding, S must agree they are both the same length.)

E slides one stick to the left about $\frac{3}{4}$ inch.

"What about now?"

"Why?"

APPENDIX B

SCORE SHEET

Name: _____ Sex: _____

Birthday: _____ Age: _____

School & Room: _____

Teacher: _____

Father's Occupation: _____

Mother's Occupation: _____

Kindergarten or Playschool Attendance: _____

I.Q.: _____

STAT 1 Score: Part 1 _____; Part 2 _____; Total _____

Conservation Score: _____

AnswersComments

1a. L, Sh. S

1b. L, B, S

2a. T, W, S

2b. $7\frac{1}{2}$, 4, S

3a. 4-4, 1-7, S

3b. L, Gr, S

4a. 1, 2, S

4b. T, W, S

5a. St, B, S

5b. L, R, S

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